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Overview of the current Dynamics of ACCORD CSC's

Presented by *Voitus Fabrice*

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Many Contributors :

Pierre Bénard, Radmila Brozkova, J.-François Geleyn, Gwenaelle Hello, Karim Yessad, Ryad Elkhatib, Sylvie Malardel, Alain Joly, Gabor Radnoti, Chris Smith, Mariano Hortal, Peter Lynch, Josef Vivoda, Petra Smolikova, Jan Masek, Filip Vana, Ludovic Auger, Piet Termonia, Daan Degrauwe, Rafiq Hamdi, Yann Seity, Steven Caluewerts, Harold Petithomme, Benjamin Mennetrier, Claire Laurent, Colm Clancy, Nika Kastelec, Léo Rogel, Louis Lutun, …..

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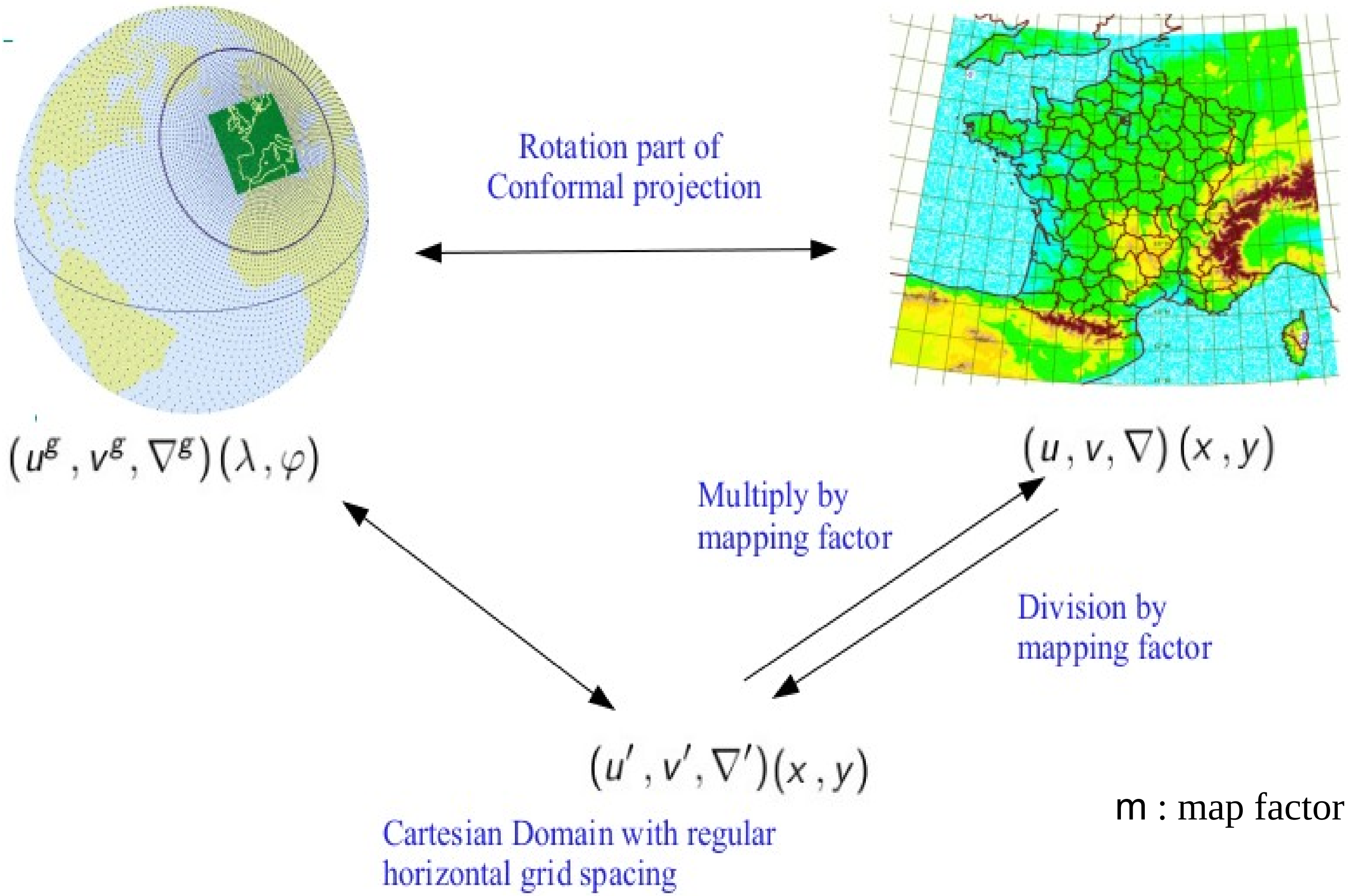
Outline

1. Horizontal Domain
2. Terrain-following Vertical coordinate
3. System of Equations
4. Space Discretizations
5. Semi-Lagrangian Time integration
6. Coupling

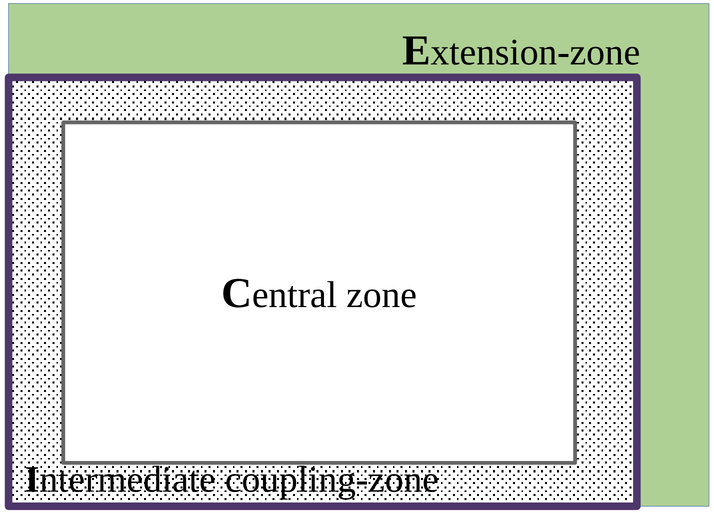
From Map Projection to Computational Domain, and vice versa

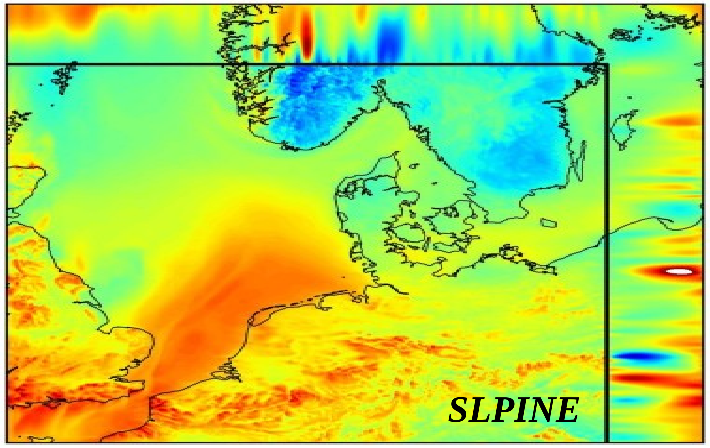


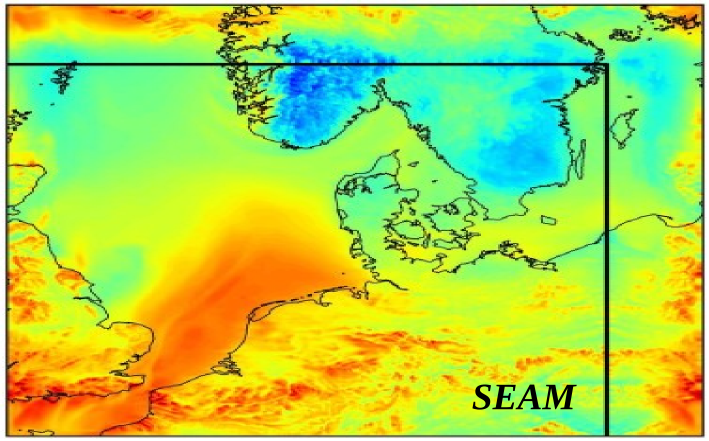
Biperiodicized Cartesian Domain

Horizontal Fields must be extended periodically to allow the use of Bi-Fourier Spectral Transform Method. It is a **Fill Gap** issue :

- **Cubic Spline** Extension method : polynomial extrapolation → may cause over- or under-shoots over complex terrain region.
- **S**ymetric **E**xtension **A**nd **M**ixing method : symetric and anti-symetric prologation are then mixed using a relaxation funtion. → less over/under shoots.
- **Boyd** Extension Method : best of all extension methods, starting from a wider C+I domain







Hydrostatic pressure-based terrain-following hybrid vertical Coordinate

- From $z \rightarrow \pi$ (hydrostatic-like pressure) :

$$\frac{\partial \pi}{\partial z} = -\rho g,$$

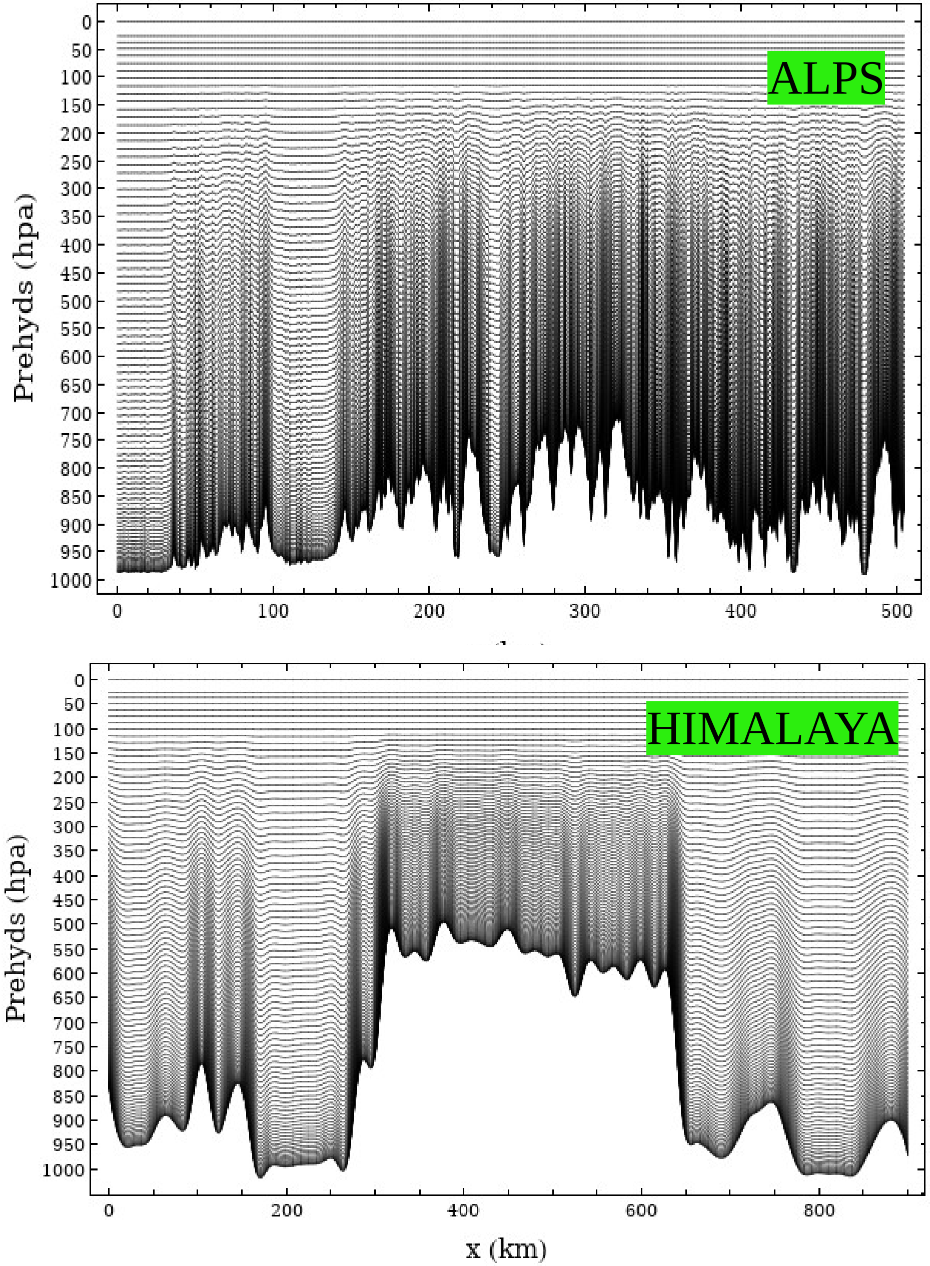
- From $\pi \rightarrow \eta$ (hybrid coordinate):

$$\pi = A(\eta) + B(\eta)\,\pi_s(x, y, t), \quad \eta \in [0, 1]$$

π_s is the hydrostatic surface pressure.
$A(\eta)$ and $B(\eta)$ are prescribed vertical functions of η only, build using piecewise differentiable polynomial functions satisfying :

- $A(0) = 0$ and $B(0) = 0$ at top $\rightarrow (\pi_T = 0)$ $\rightarrow z_T$ is repulsed to infinity.
- $A(1) = 0$, $B(1) = 1$ at bottom $\rightarrow \pi = \pi_s$
- monotonicity constraint :

$$(\mathrm{d}A/\mathrm{d}\eta) + (\mathrm{d}B/\mathrm{d}\eta)\pi_s > 0$$



Implications of such a vertical coordinate :

- For a scalar variable ψ, and the wind-vector $\mathbf{V} = (\mathbf{v}, \mathrm{w})$,

$$\frac{\partial \psi}{\partial z} = \frac{\partial_\eta \psi}{\partial_\eta z}$$

$$\nabla \psi = \nabla_\eta \psi - \nabla_\eta z \, \frac{\partial_\eta \psi}{\partial_\eta z}$$

$$\boldsymbol{\nabla} \cdot \mathbf{V} = \nabla_\eta \cdot \mathbf{v} + \frac{\partial_\eta \mathrm{w}}{\partial_\eta z} - \nabla_\eta z \cdot \frac{\partial_\eta \mathbf{v}}{\partial_\eta z}$$

- Vertical and horizontal metrics terms are given by

$$\partial_\eta z = -\frac{\partial_\eta \pi}{\rho g},$$

$$\nabla_\eta z = \nabla_\eta z_S + \int_\eta^1 \nabla_\eta \left(\frac{\partial_\eta \pi}{\rho g} \right) \mathrm{d}\eta'$$

- Metrics $\partial_\eta z$, and $\nabla_\eta z$ are time-dependent.
- Imply to computation of discrete vertical integral operators as :

$$\mathcal{G}(\psi) = \int_\eta^1 \psi \, \frac{\partial_\eta \pi}{\pi} \mathrm{d}\eta'$$

$$\mathcal{S}(\psi) = \frac{1}{\pi} \int_0^\eta \psi \, \partial_\eta \pi \mathrm{d}\eta'$$

$$\mathcal{N}(\psi) = \frac{1}{\pi_s} \int_0^1 \psi \, \partial_\eta \pi \mathrm{d}\eta'$$

- Extension to Non-hydrostatic (NH) system can be sought as an **ADD ON** to the existing Hydrostatic Model, by introducing the NH pressure departure :

$$\hat{q} = \ln (p/\pi)$$

Doubly-Blended Dry and Abdiabatic Dynamical systems

***(δh , δq)** couple of switches for various systems HPE (0,1), QE (1,0), EE (1,1) :*

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -2\left(\Omega \times \mathbf{v}\right) - \frac{RT}{\pi}\nabla_\eta \pi - \mathrm{g}\nabla_\eta z - RT\nabla_\eta \hat{q} - \mathrm{g}\frac{\partial_\eta\left(p^{\kappa_q} - \pi^{\kappa_q}\right)}{\partial_\eta \pi^{\kappa_q}}\nabla_\eta z$$

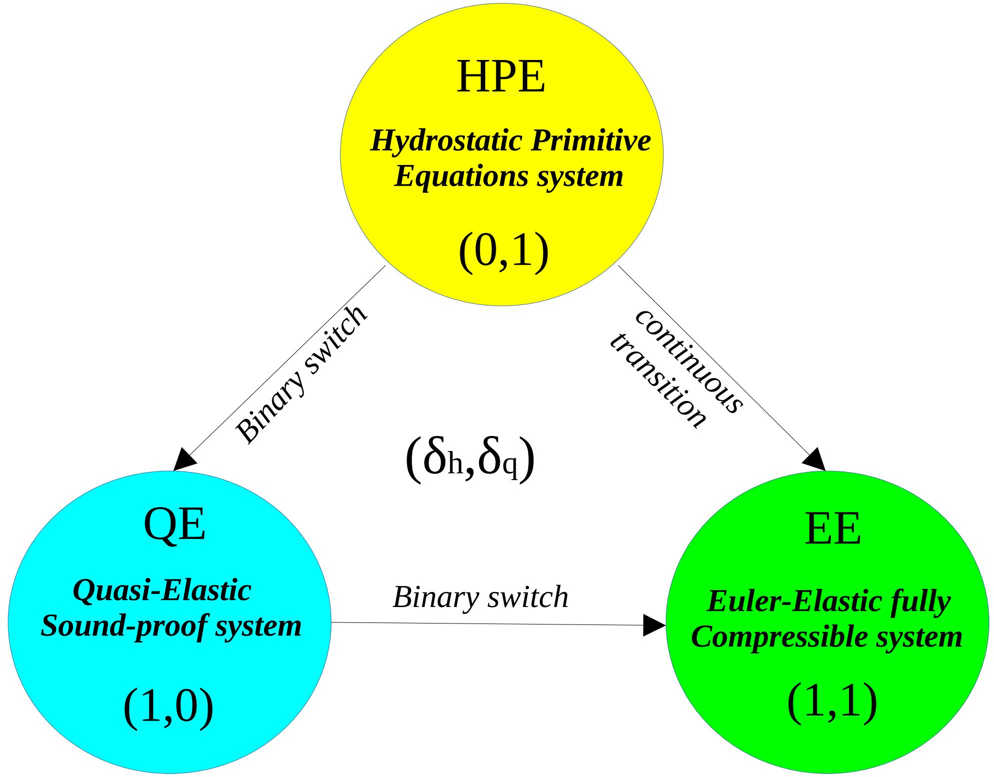
$$\frac{\mathrm{D}w}{\mathrm{D}t} = \mathrm{g}\frac{\partial_\eta\left(p^{\kappa_q} - \pi^{\kappa_q}\right)}{\partial_\eta \pi^{\kappa_q}}$$

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \frac{RT}{C_p}\left[\left(1 - \delta_q\delta_h\right)\frac{\dot{\pi}}{\pi} - \delta_q\delta_h\frac{C_p}{C_v}\boldsymbol{\nabla}\cdot\mathbf{V}\right]$$

$$\delta_q\frac{\mathrm{D}\hat{q}}{\mathrm{D}t} = -\delta_h\left[\frac{\dot{\pi}}{\pi} + \frac{C_p}{C_v}\boldsymbol{\nabla}\cdot\mathbf{V}\right]$$

$$\frac{\mathrm{D}\pi_s}{\mathrm{D}t} = -\mathbf{v}\cdot\nabla_\eta\pi_s - \int_0^1 \nabla\left(\partial_\eta \pi\,\mathbf{v}\right)\mathrm{d}\eta'$$

with $\kappa_q = \delta_q + (R/C_p)(1 - \delta_q)$.

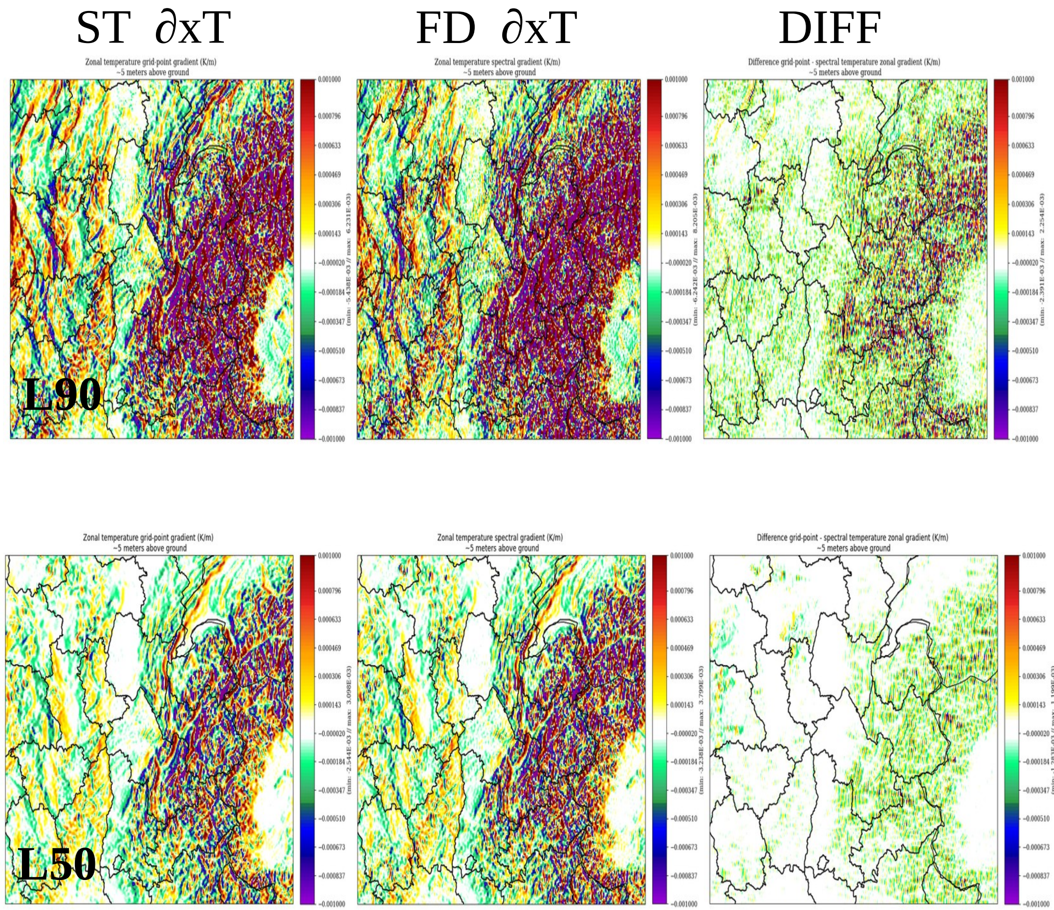


- Lagrangian time derivative :

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_\eta + \dot{\eta}\partial_\eta$$

- $\dot{\pi}$ and $\dot{\eta}$ are diagnosed by vertically integrating the hydrostatic pressure-based formulation of the mass-continuity equation.

Horizontal discretization



- All felds are co-located on the horizontal grid (A-Grid)

- Horizontal derivatives along constant η-level are discretely computed via :

  - Bi-Fourier **S**pectral **T**ransform (**ST**) method.

  - Grid-point local second order accurate **F**inite-**D**ifferences (**FD**), or **F**inite-**V**olumes (**FV**) methods (as an option since cy49t2) using : **SL HALO** approach or **ATLAS** library

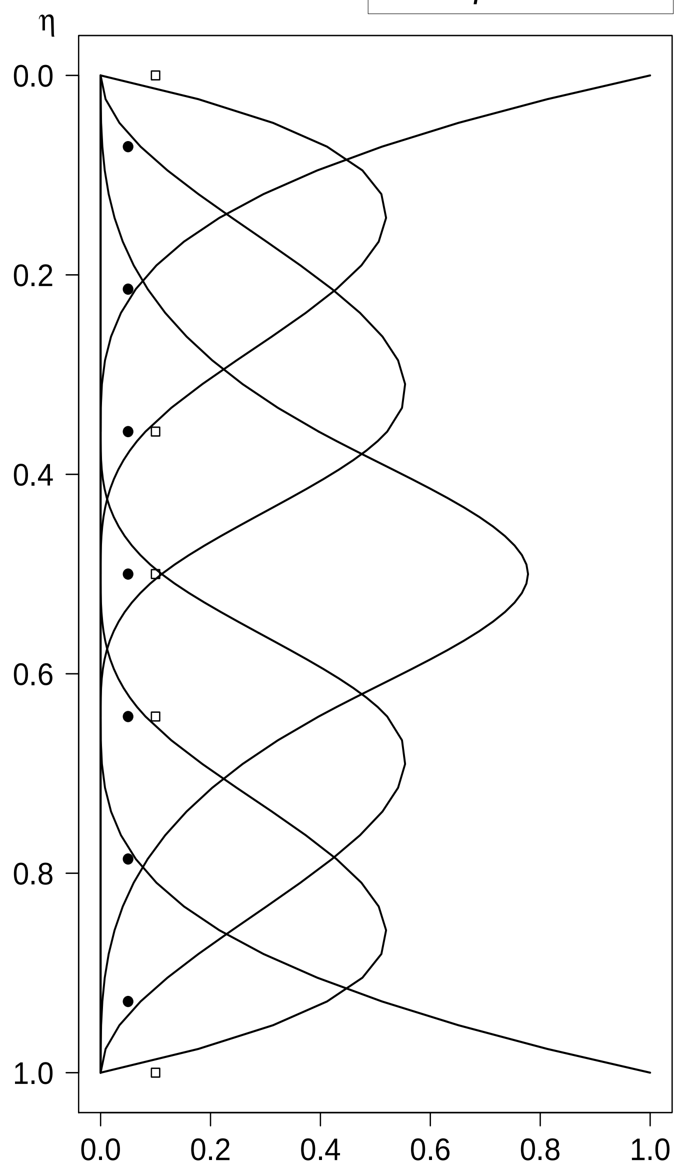
Vertical Discretization

- A vertical Lorenz grid is used : π, w, and z are placed at the interfaces (half-levels), the remaining variables are on the layers (full-levels).

- Vertical derivatives and integrals involved in the dynamical system are discretely computed via:

  - Second-order **V**ertical **F**inite-**D**ifferences (**VFD**) method.
  - High-order **V**ertical **F**inite-**E**lement (**VFE**) Galerkin method.

VFE cubic spline basis functions



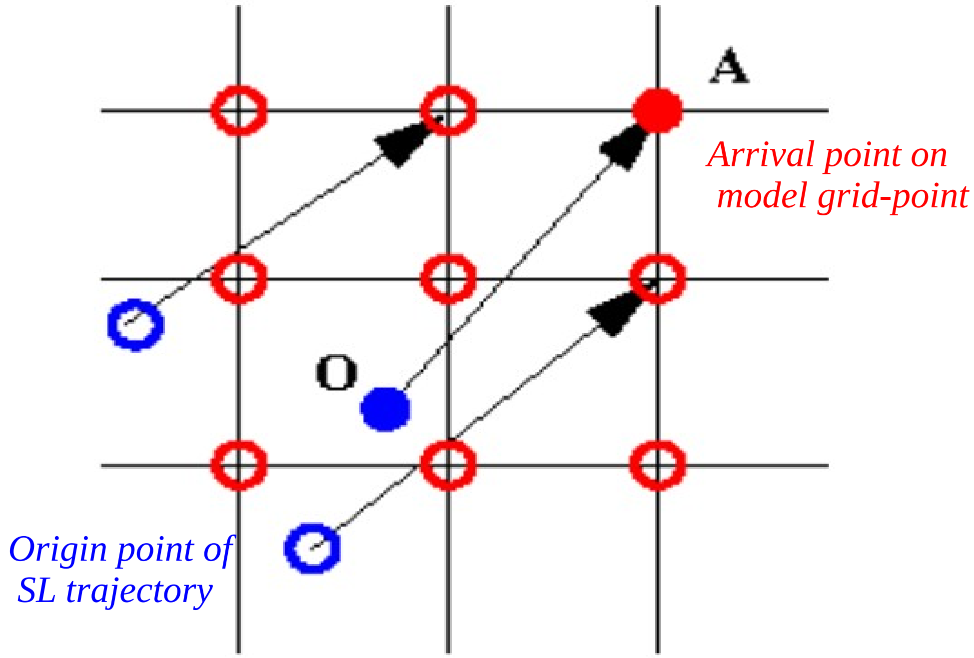
Semi-Lagrangian « pointwise » Time integration

- Denoting by $\mathbf{x} = (x, y, \eta)$ the location of the fluid particle, and $\mathcal{X}$ the state-vector of prognostic variables. In Lagrangian formalism, the coupled system to be integrated in time writes as :

$$\frac{\mathrm{D}\mathbf{x}}{\mathrm{D}t} = \dot{\mathbf{x}} \qquad \text{(Kinematics)}$$

$$\frac{\mathrm{D}\mathcal{X}}{\mathrm{D}t} = \mathcal{D}(\mathcal{X}) \qquad \text{(Dynamics)}$$

- $\dot{\mathbf{x}} = (\dot{x}, \dot{y}, \dot{\eta})$: contra-variant wind-vector components.
- $\mathcal{D}$: resolved non-linear dynamical processes (e.g, wave-propagation).



« Ideal » SL time integration scheme

- Trapezoidal implicit centered time scheme :

$$\mathbf{x}_O = \mathbf{x}_A - \Delta t \left(\frac{\dot{\mathbf{x}}_A^+ + \dot{\mathbf{x}}_O^0}{2} \right)$$

$$\mathcal{X}_A^+ = \mathcal{X}_O^0 + \Delta t \left(\frac{\mathcal{D}(\mathcal{X}^+)_A + \mathcal{D}(\mathcal{X}^0)_O}{2} \right)$$

- $(0, +) \rightarrow (t, t + \Delta t)$.

SL Iterative Centered Implicit (ICI) Algorithm

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ICI QUASI-NEWTON OUTER LOOP

FIXED-POINT INNER LOOP

Initial guess for prognostic variables : $\mathcal{X}_A^{+(-1)} \leftarrow \mathcal{X}_A^0$

Initial guess for transporting velocities : $\dot{\mathbf{x}}_A^{+(-1)} \leftarrow \dot{\mathbf{x}}_A^0$

for $k = 0$ to NSITER **do**

Initial guess for trajectory research : $\mathbf{x}_{O_k^{(0)}} \leftarrow \mathbf{x}_A$

for $\nu = 1$ to NITMP **do**

Compute weight for interpolations at $O_k^{(\nu-1)}$

Trajectory research : $\mathbf{x}_{O_k^{(\nu)}} \leftarrow \mathbf{x}_A - \frac{\Delta t}{2}\left(\dot{\mathbf{x}}_A^{+(k-1)} + \dot{\mathbf{x}}^0_{O_k^{(\nu-1)}}\right)$

end for

Compute weights for interpolations at $O_k = O_k^{(\text{NITMP})}$

Compute explicit guess : $\widetilde{\mathcal{X}}_A^{+(k)} \leftarrow \left(\mathcal{X}^0 + \frac{\Delta t}{2}\mathcal{D}\left(\mathcal{X}^0\right)\right)_{O_k} + \frac{\Delta t}{2}\mathcal{D}\left(\mathcal{X}^{+(k-1)}\right)_A$

Add explcit linear correction : $\widetilde{\mathcal{X}}_A^{+(k)} \leftarrow \widetilde{\mathcal{X}}_A^{+(k)} - \frac{\Delta t}{2}\mathcal{L}^*.\mathcal{X}_A^{+(k-1)}$

Solve implicit linear system : $\left(I - \frac{\Delta t}{2}\mathcal{L}^*\right).\mathcal{X}_A^{+(k)} = \widetilde{\mathcal{X}}_A^{+(k)}$

end for

- **Ultimate Goal** : Converge iteratively towards the ideal **Trapezoidal implicit SL scheme**.

- Convergence of the fixed-point iterative SL research algorithm is ensured for local Lipschitz number

$$\ell = \Delta t \|\|(\partial\dot{\mathbf{x}}/\partial\mathbf{x})\|\|$$

smaller than 1.

- $\mathcal{L}^*$ is a linear differential operator judiciously chosen to ensure **stability** and **convergence** of the ICI algorithm $\rightarrow$ Does not need to be the proper linear-tangent of $\mathcal{D}$.

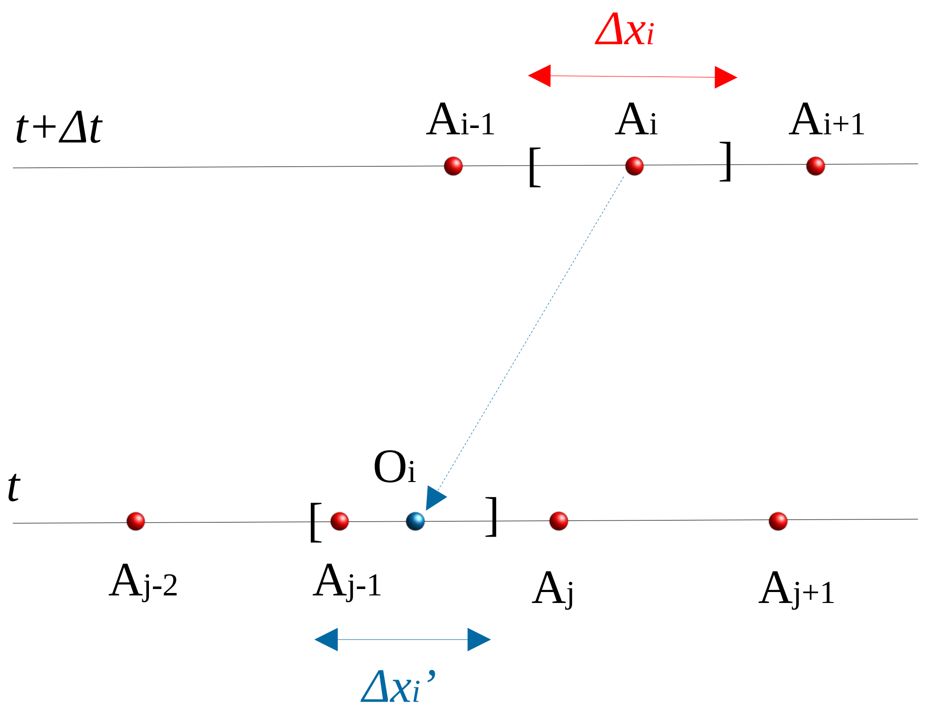
Lagrangian Interpolations at Origin point

(Linear/Quadratic/Cubic) Lagrangian interpolations at departure point with the following options :

- **SLHD** : Aim at controlling the inherent diffusive property of the SL interpolations taking into account the horizontal flow deformation tensor. (Need for specific setting for each transported variable).

- **COMAD** : Improved the conservative property of the SL-interpolations taking into account the deformation of the air parcell surrounding the origin point along each direction. (Linear interp. for hydrometeors, cubic for others)

- **SWEEP** : Alternated quadratic interpolations from one time-step to an other (better conservative property), cf Petra's talk.



Monotic preservation : Clipping limiters (LQMH, LQMV, LQM3D), vertical mass redistribution (based on ILMC approach, in preparation)

Variable conversion for implicit correction

Suitable variable conversion helps to get more stable and convergent algorithm

SL treatment of Coriolis term (δ_V=1)

SL orographic resonance Treatment (δ_R=1)

Advected prognostic variables — $\mathcal{X}$

- $\mathbf{v} + \delta_V(\Omega \times \mathbf{x})$
- w
- $T + \delta_R \dfrac{\Phi_s}{R_d T_s^{ref}} \alpha_T^{ref}(\eta)$
- $\hat{q}$
- $\ln(\pi_s) + \delta_R \dfrac{\Phi_s}{R_d T_s^{ref}}$

$\mathcal{T}$ / $\mathcal{T}^{-1}$

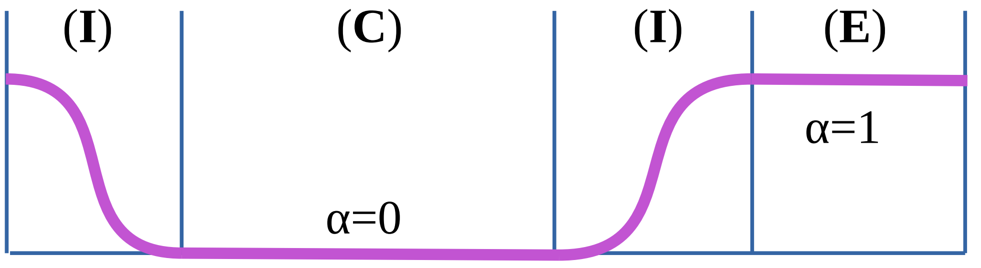
Implicitly corrected prog. variables — $\mathcal{Z}$

- $\mathbf{v}' = m\mathbf{v}$
- $d = \dfrac{\partial_\eta w}{\partial_\eta z} - \nabla_\eta z \cdot \dfrac{\partial_\eta \mathbf{v}}{\partial_\eta z}$
- $T_v = T\left(\dfrac{R}{R_d}\right)$
- $\hat{q}$
- $\ln(\pi_s)$

Implicit linear system is solved for $\mathcal{Z}$ vector-state by Converting $\mathcal{X}$-explicit guesses

$$\left(I - \frac{\Delta t}{2}\mathcal{L}^*\right).\mathcal{Z}_A^{+(k)} = \mathcal{T}\left(\widetilde{\mathcal{X}}_A^{+(k)}\right) - \frac{\Delta t}{2}\mathcal{L}^*.\mathcal{Z}_A^{+(k-1)}$$

How Lateral Coupling is performed ?



α : *relaxation function varying gradually from 0 in the C-zone to 1 in the E-zone.*

Pre-requisite due to periodicity constraint :

- The Large-Scale Lateral boundary conditions fields $\mathcal{Z}^{+}_{LS}$ must be extended periodically in the **E**xtension zone.

Coupling procedure :

- Add explcit linear correction to the explicit guesses transformed into $\mathcal{Z}$ state-vector space :

$$\widetilde{\mathcal{Z}}^{+(k)} = \mathcal{T}\left(\widetilde{\mathcal{X}}^{+(k)}\right) - \frac{\Delta t}{2}\mathcal{L}^{*}.\mathcal{Z}^{+(k-1)}$$

- **Relaxation** of explicit RHS towards its Large-Scale (LS) counterpart is performed as follow :

$$\widetilde{\mathcal{Z}}^{C} = (1-\alpha)\widetilde{\mathcal{Z}}^{+(k)} + \alpha\left(\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^{*}\right).\mathcal{Z}^{+}_{LS}$$

- Solve implicit linear system with coupled explcit RHS:

$$\left(\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^{*}\right).\mathcal{Z}^{+(k)} = \widetilde{\mathcal{Z}}^{C}$$

SI linear blended operator, and Solver for the linear implicit system

(Tr^*, Ta^*, πs^*) **are stabilizing SI parameters** characterizing the linear operator $\mathcal{L}^*$:

1. (Tr^*, Ta^*, πs^*) **are assumed to be constant in space and time** → current status in All CSC's due the use of (**ST**) method → Spectral direct Solver can be applied. → **potential stability or convergence issue** over steep slopes/high-plateau montainuous regions.

2. (Tr^*, Ta^*, πs^*) **are taken horizontally homogeneous but varying vertically and in time** → recently developed **SI-UPDATE** approach → flow-dependent → more robust and spectral can still be applied.

3. (Tr^*, Ta^*, πs^*) **are space and time dependent** → **moving away from (ST) method**. → Need to investigate Krylov preconditioned or Multigrids Solvers

- Blended linear system $(I - \frac{\Delta t}{2}\mathcal{L}^*)\mathcal{Z} = \widetilde{\mathcal{Z}}^C$ to be solved at each outer iteration :

$$\mathbf{v}' + \frac{\Delta t}{2}\nabla'_\eta \Big[\mathcal{G}^*(R_d T_v) + T_s^* \ln(\pi_s) + (I - \mathcal{G}^*)(R T_r^* \hat{q}) \Big] = \widetilde{\mathbf{v}'}^C$$

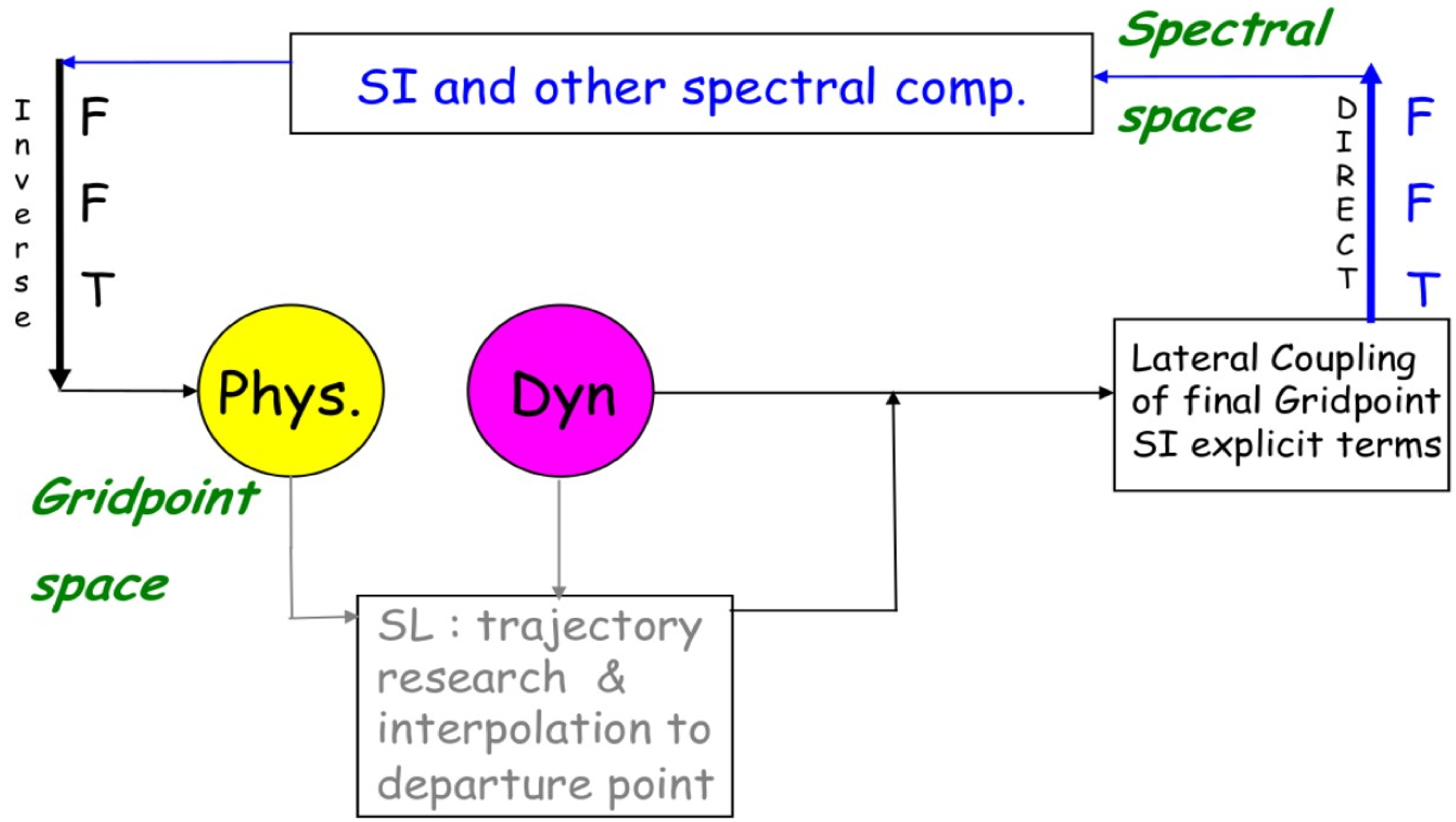
$$d + \frac{\Delta t}{2}\frac{g^2}{R T_a^*}\mathcal{L}^*_{\kappa_q}(\hat{q}) = \widetilde{d}^C$$

$$T_v + \frac{\Delta t}{2}\frac{R T_r^*}{C_p}\Big[(1 - \delta_q\delta_h)\mathcal{S}^*(D') + \delta_q\delta_h \frac{C_p}{C_v}\left(D' + d\right) \Big] = \widetilde{T_v}^C$$

$$\delta_q \hat{q} + \delta_h \frac{\Delta t}{2}\frac{C_p}{C_v}\Big[D' - \frac{C_v}{C_p}\mathcal{S}^*(D') + d \Big] = \widetilde{\hat{q}}^C$$

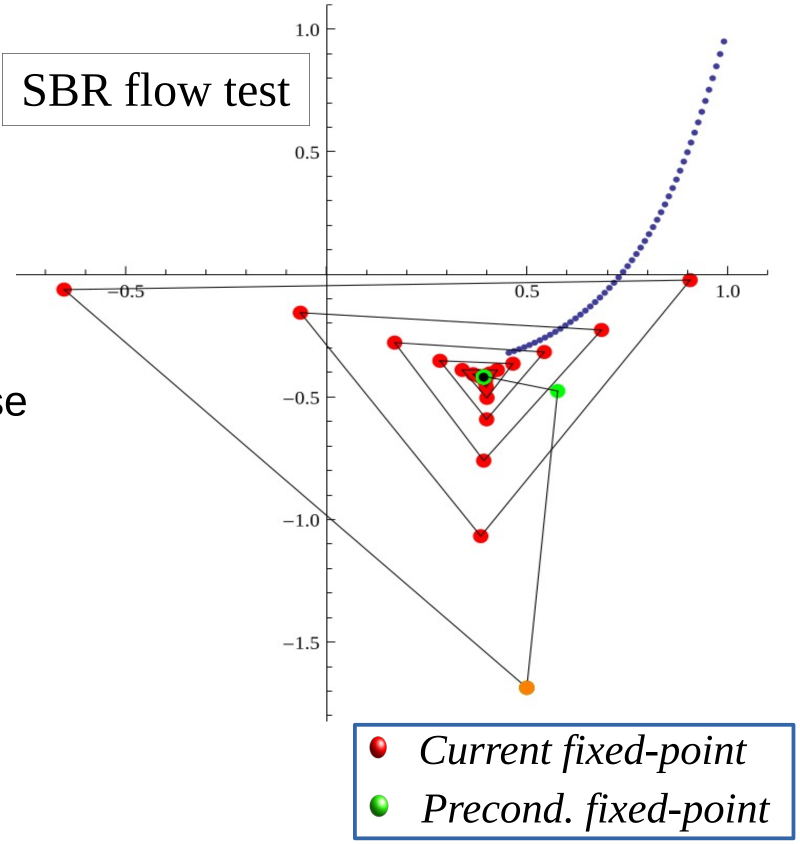
$$\ln(\pi_s) + \frac{\Delta t}{2}\mathcal{N}^*(D') = \widetilde{\ln(\pi_s)}^C$$

A brief Schematic Summary of the Model time loop



On-Going Activities & Plans

- **Improve Departure point SL research** : Speed-up the convergence of the fixed-point method for large Lipchitz number at high-resolution, by (i) taking into account a smarter starting point, (ii) precondioning the reseach through jacobian of the flow.



- **High-order COMAD interpolations** : refined higher-order piecewise re-construction with a better conservative and shape-preserving properties of the COMAD SL-interpolations

- **Towards a more robust SI linear system** :

(a) Investigate potential increase of robustness brings by SI-UPDATE strategy for the current direct spectral solver.

(b) Design of a horizontally varying coeff. SI linear system with Krylov GCR(k) or Multigrids solver for more stability over steep slopes → Replace (**ST**) derivatives by local grid-point horizontal discretization (**FD**/**FV**) allowed by recent SL-HALO or ATLAS approach.

- **Plan to be revisited :** orography filtering, horizontal numerical diffusion, vertical spectral nudging, upper absorbing layer, in the context of very high resolution

Documentation are on their way

A more comprehensive scientific documentation is in preparation, as well as a user guide documentation describing the various namelist keys and related options supported by the current Dynamics. [Cf. Jana's Talk]

Thanks for your attention !