Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

ACCORD
A Consortium for COnvection-scale modelling
Research and Development

News in dynamics ~~and coupling~~ for high resolution applications

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GeoSphere Austria · DHMZ · Czech Hydrometeorological Institute · HungaroMet · IMGW · METEO ROMANIA · SHMÚ · ARSO METEO Slovenia

Outline

Topics recently being solved in LACE:

- Sweep interpolations - Natalia Szopa
- Future dynamics setting in C-LAEF AlpeAdria - Clemens Wastl, Endi Keresturi
- Documentation for dynamics - Nika Kastelec
- Higher vertical resolution and implications in the turbulence scheme - Mario Hrastinski talk in Physics session

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ACCORD dynamical core

- Iterative centered implicit (ICI) time scheme: linearization of basic equations around a reference state followed by implicit treatment of linear part and iterative procedure for non-linear residual

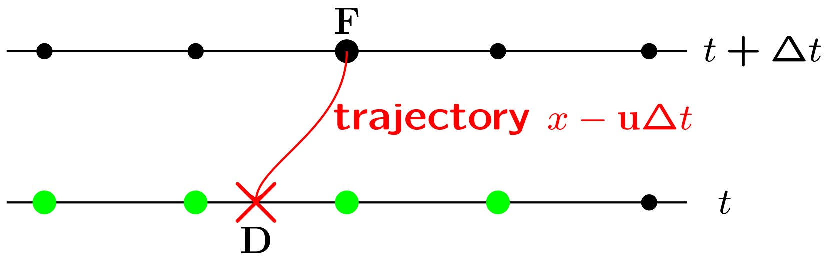
- The choice of model variables to minimalize the non-linear residual of the ICI scheme

- Semi-Lagrangian advection

- Transform method: Helmholtz elimination for semi-implicit part of the time scheme and horizontal diffusion are solved in spectral space, while advection and physical parametrizations in grid-point space; direct and inverse transformations happens every time step

Semi-Lagrangian advection

- numerically efficient transport scheme
- removes the obvious non-linearity associated with advection
- allows for long time-steps and linear truncation
- the damping associated with interpolations may be controlled and seen as a desirable form of numerical diffusion that suppresses small-scale noise
- conservativity not guaranteed



Necessary steps:

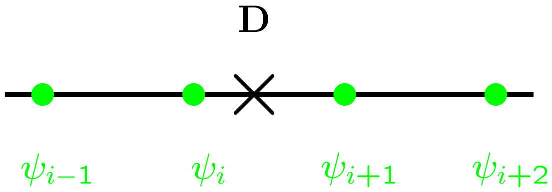
- trajectory search to get departure points D
- interpolation to departure points
- advection of quantity - simple $\psi_F(t + \Delta t) = \psi_D(t)$

COMAD (COntinuous Mapping about Departure points) correction may be applied to the standard interpolation weights taking into account the deformation of the air parcels along each direction in order to improve the continuity and the conservative property of the re-mapping between the model grid points and the departure points of the parcels trajectories

Interpolation order and cost

- advected scalar ψ in one dimension $\frac{d\psi}{dt} = 0$
- equidistant grid-points with the distance Δx
- 4 grid-points neighborhood is available

centered interpolation



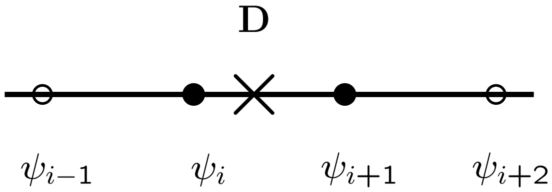
$$\psi_D = c_{i-1}\psi_{i-1} + c_i\psi_i + c_{i+1}\psi_{i+1} + c_{i+2}\psi_{i+2} + E_c$$

$E_c \approx \mathcal{O}(\Delta x^4)$, fourth order accuracy

computational cost

$\approx 4^2$ multiplications

linear interpolation



$$\psi_D = l_i\psi_i + l_{i+1}\psi_{i+1} + E_l$$

$E_l \approx \mathcal{O}(\Delta x^2)$, second order accuracy

computational cost

$\approx 2^2$ multiplications

The sweep interpolation method

- **Mortezazadeh and Wang (2017) first described "sweep" interpolation as a method that reduces both dissipation and dispersion errors**

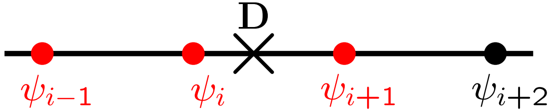
  Mortezazadeh, M. and Wang, L.: A high-order backward forward sweep interpolating algorithm for semi-Lagrangian method, Int. J. Num. Meth. Fluids, 84, 584–597, https://doi.org/10.1002/fld.4362, 2017.

  Mortezazadeh, M., Cossette, J.-F., Dastoor, A., de Grandpré, J., Ivanova, I., and Qaddouri, A.: Sweep interpolation: a cost-effective semi-Lagrangian scheme in the Global Environmental Multiscale model, Geosci. Model Dev., 17, 335–346, https://doi.org/10.5194/gmd-17-335-2024, 2024.

- **it performs SL interpolation with the same computational cost as a third order polynomial scheme but with the accuracy of a fourth order interpolation scheme**
- **it was implemented in ACCORD LAM (CY48T3) based on Filip Váňa's implementation in IFS**

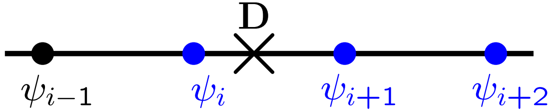
The sweep interpolation method

backward interpolation using 3 grid – points applied in even time – steps



$$\psi_D = a_{i-1}\psi_{i-1} + a_i\psi_i + a_{i+1}\psi_{i+1} + E_b$$

forward interpolation using 3 grid – points applied in odd time – steps



$$\psi_D = b_i\psi_i + b_{i+1}\psi_{i+1} + b_{i+2}\psi_{i+2} + E_f$$

compensating errors in the consecutive time steps

$$E_b = -c\frac{\partial^3\psi_D}{\partial x^3} + \mathcal{O}(\Delta x^4)$$

$$E_f = c\frac{\partial^3\psi_D}{\partial x^3} + \mathcal{O}(\Delta x^4)$$

after two time steps

$$E_b + E_f \approx \mathcal{O}(\Delta x^4)$$

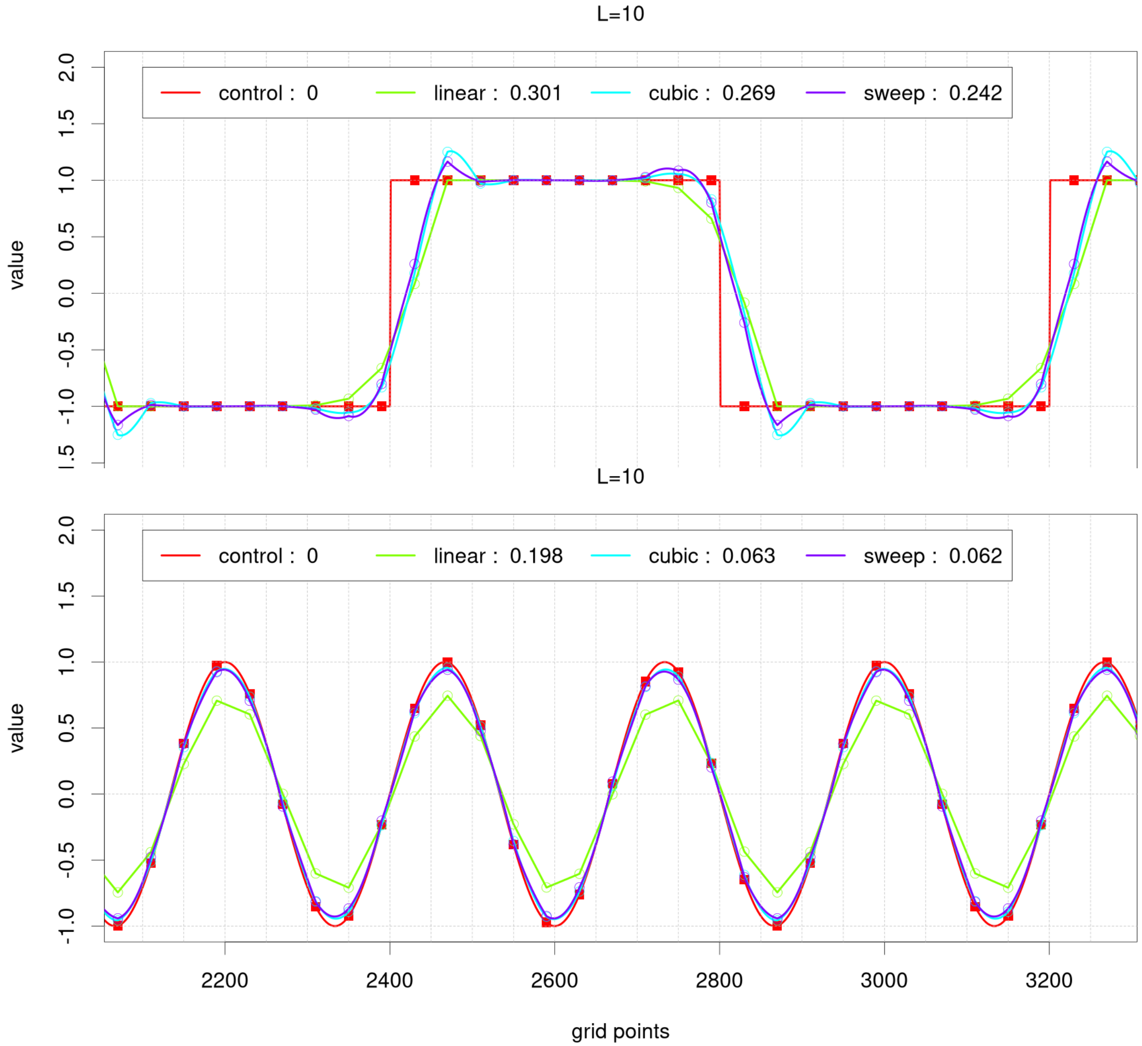
computational cost $\approx 3^2$ multiplications

computational savings $\approx 1 - \left(\frac{3}{4}\right)^2 \approx 40\%$

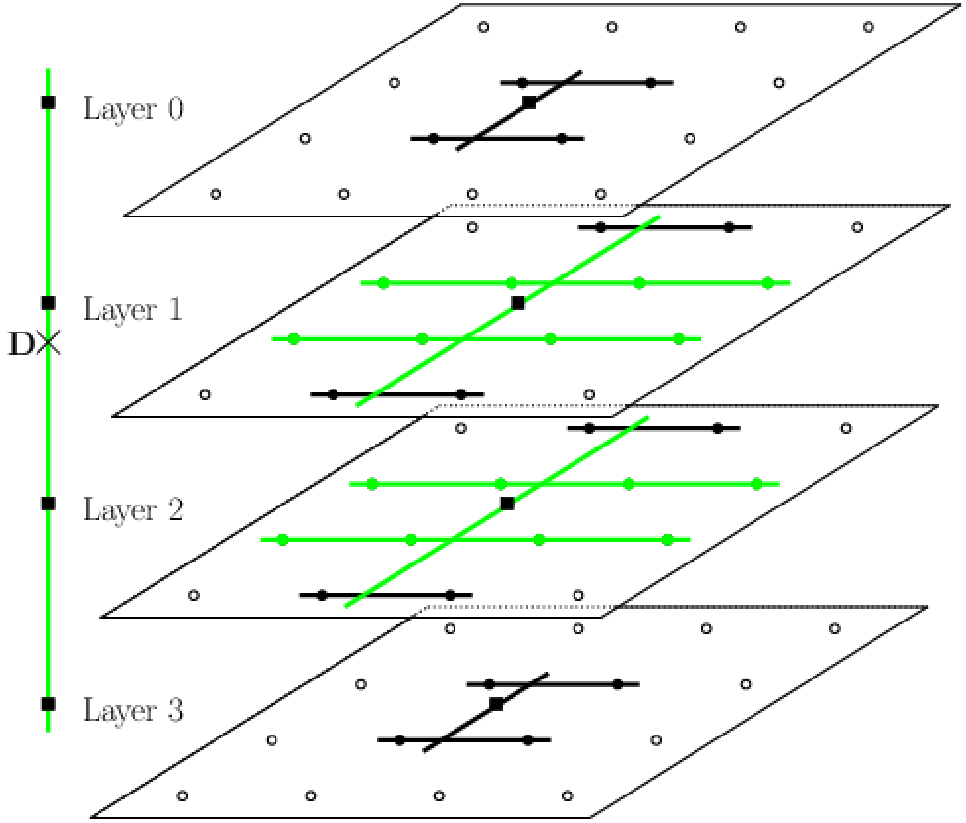
compared to cubic Lagrange

Interpolation of simple 1D function

- we simulate the behavior of various interpolations on the regular advection of one-dimensional simple functions in the form of regular square or a harmonic wave
- we start with the red curve known in the red squared points
- we alternate between the interpolation step and the advection step
- after 10 iterations we measure the root mean square difference from the initial red curve
- we follow the advection, thus ideally we obtain the same picture as at the beginning - the red curve



Semi-Lagrangian advection



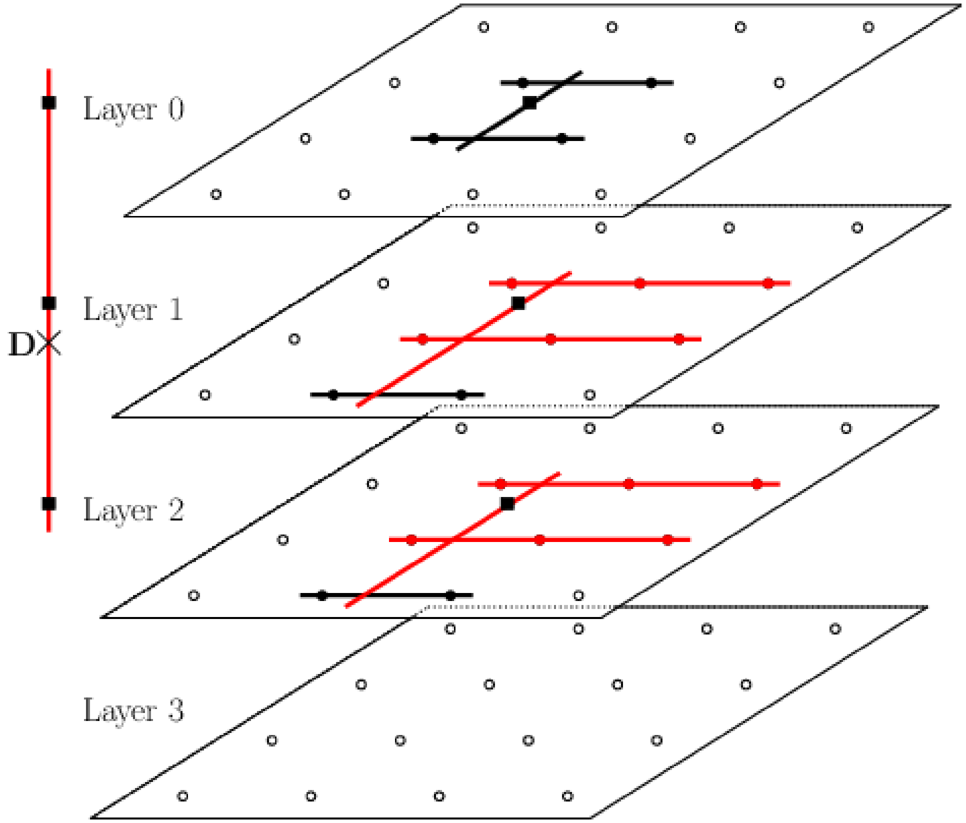
ACCORD cubic interpolation to the departure point D:

7 cubic interpolations (each using 4 grid-points)

and 10 linear interpolations (each using 2 grid-points)

32 grid-points involved

Semi-Lagrangian advection



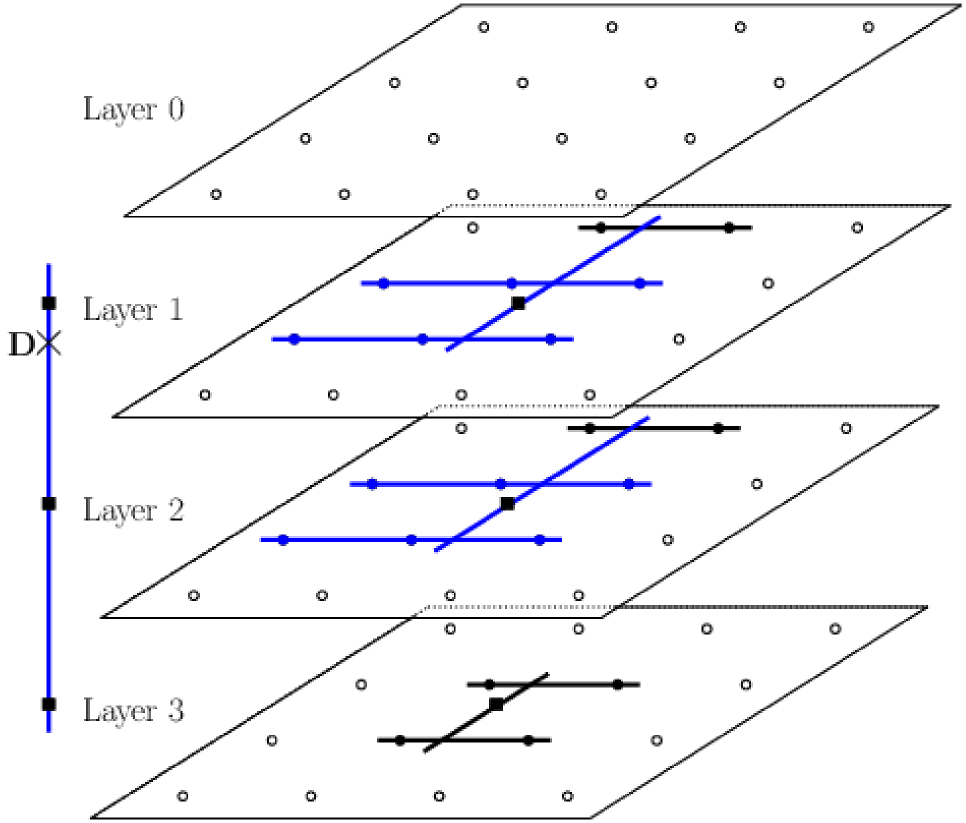
SWEEP interpolation
to the departure point D
at odd time steps:

7 quadratic interpolations
(each using 3 grid-points)

and 5 linear interpolations
(each using 2 grid-points)

20 grid-points involved

Semi-Lagrangian advection



SWEEP interpolation
to the departure point D
at even time steps:

7 quadratic interpolations
(each using 3 grid-points)

and 5 linear interpolations
(each using 2 grid-points)

20 grid-points involved
at different stencil then
in the odd time steps

Semi-Lagrangian horizontal diffusion - SLHD

Grid-point part of SLHD

- calculate the flow deformation $\mathcal{D}$ and the flow characteristic κ as a function of $\mathcal{D}$
- design an accurate interpolation operator $\mathcal{I}_A$ and a diffusive interpolation operators $\mathcal{I}_D$
- combine them with the weight κ as $(1-\kappa)\mathcal{I}_A + \kappa\mathcal{I}_D$
- do that in all three dimensions separately
- make it more complicated with several namelist parameters ;-)

Add spectral diffusion close to model top for all spectral model variables and very scale selective spectral diffusion in the whole domain for the wind variables (vorticity, divergence and vertical divergence)

How to design the interpolation operators?

Classical solution:
$\mathcal{I}_A$ is the cubic Lagrange operator
$\mathcal{I}_D$ is a quadratic 4-points operator
possibly with the Laplacian smoother
(horizontal and vertical)
controlled with parameters ϵ_H, ϵ_V

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possibly with the Laplacian smoother
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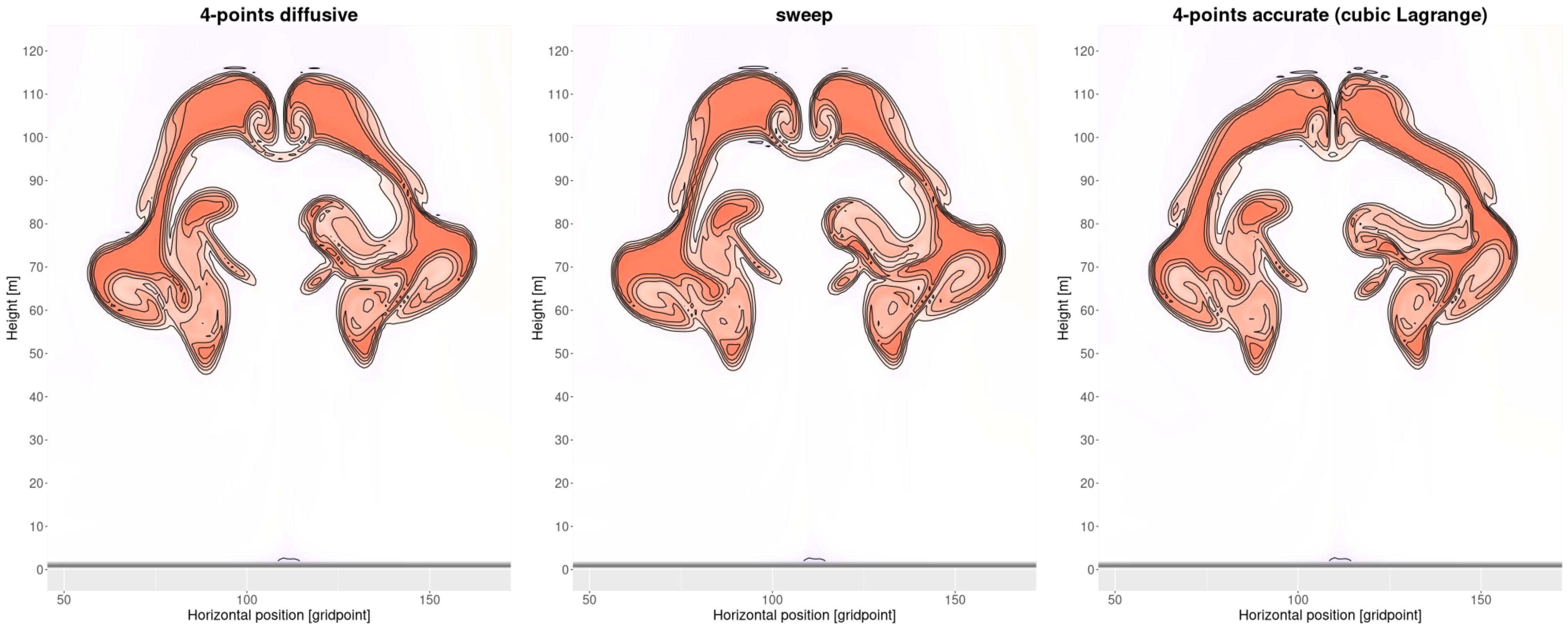
controlled with parameters ϵ_H, ϵ_V

Sweep solution:
$\mathcal{I}_A$ is the sweep operator
$\mathcal{I}_D$ is the sweep operator
with the Laplacian smoother
(horizontal and vertical)
controlled with parameters $\epsilon_H \neq 0, \epsilon_V$

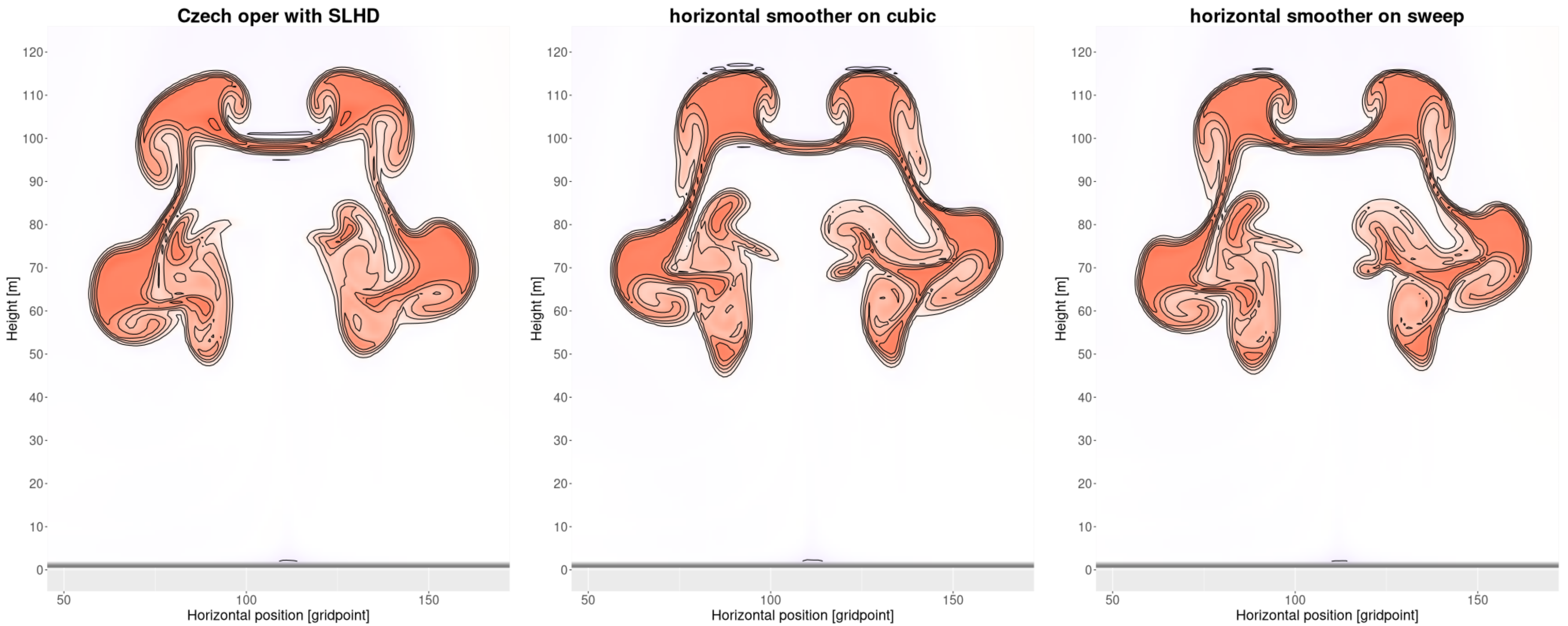
Rising warm bubble



forth order accurate

forth order accurate

Rising warm bubble



SLHD combining accurate and diffusive operator with horizontal smoother

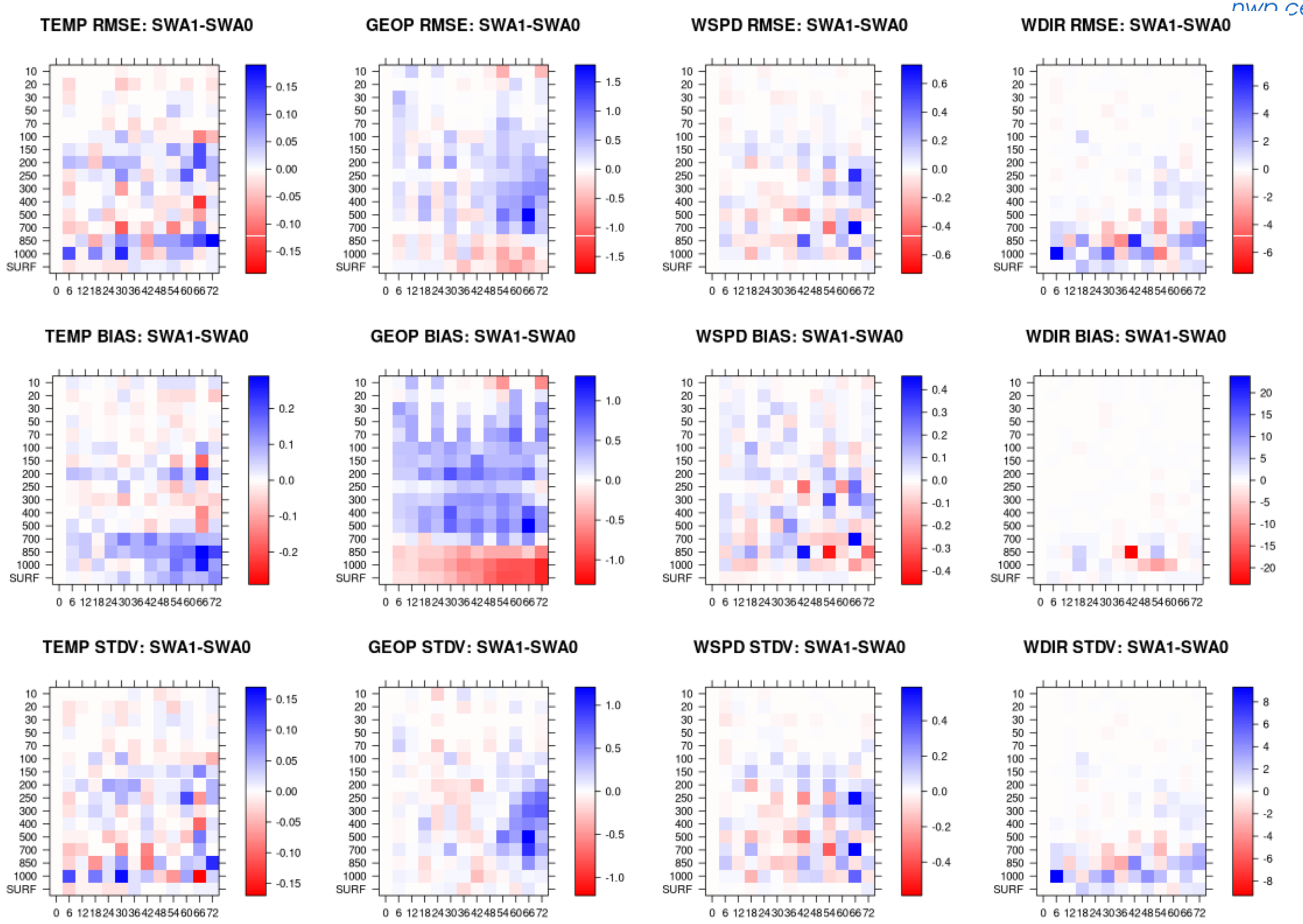
SLHD combining cubic and cubic operator with horizontal smoother

SLHD combining sweep and sweep operator with horizontal smoother

Real simulation with SLHD

SWEEP SLHD
- CUBIC SLHD

18 - 31 Oct 2025
Czech domain
including Alps



RMSE

BIAS

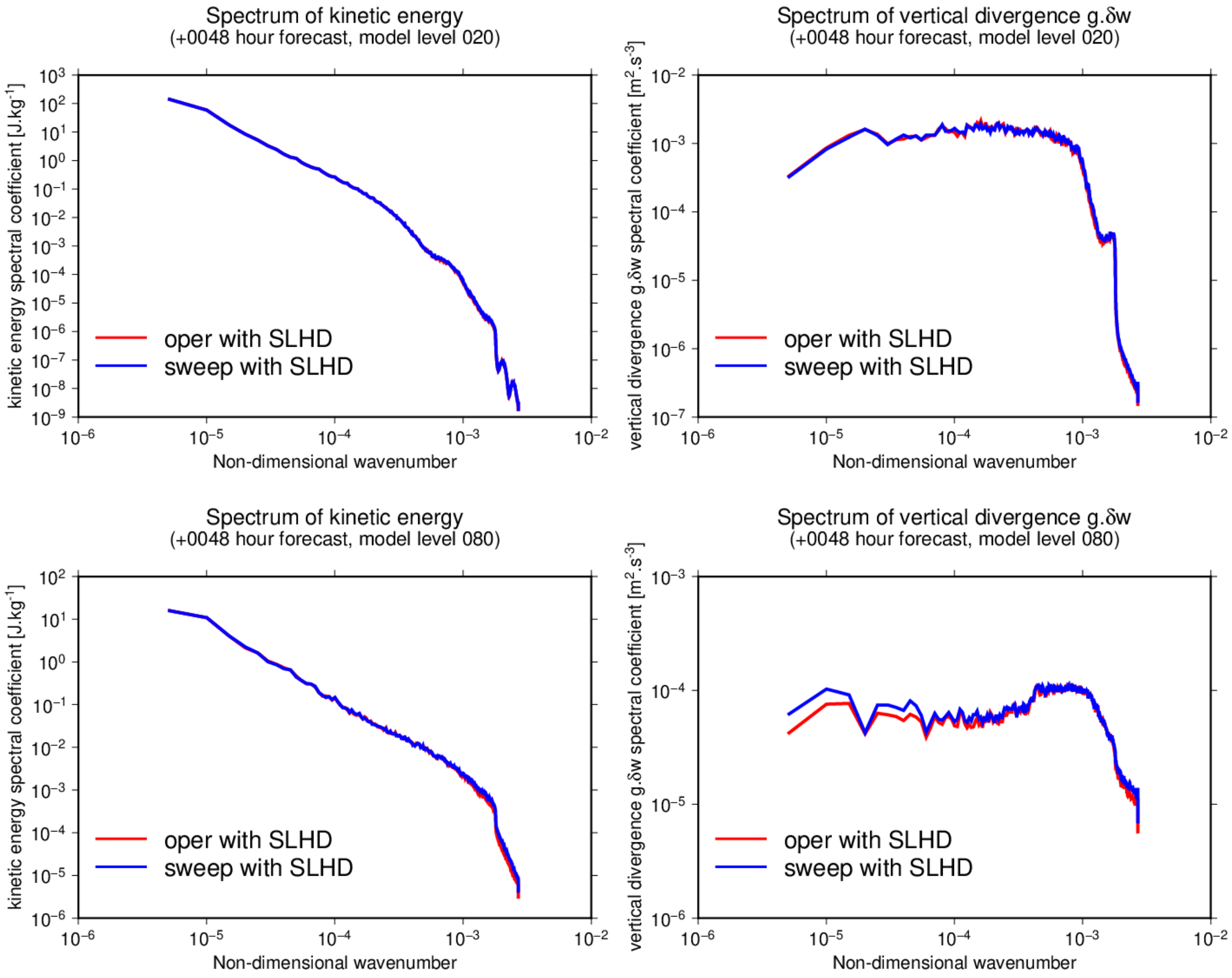
STDEV

TEMPER GEOPOT WIND SPEED WIND DIR

Real simulation with SLHD

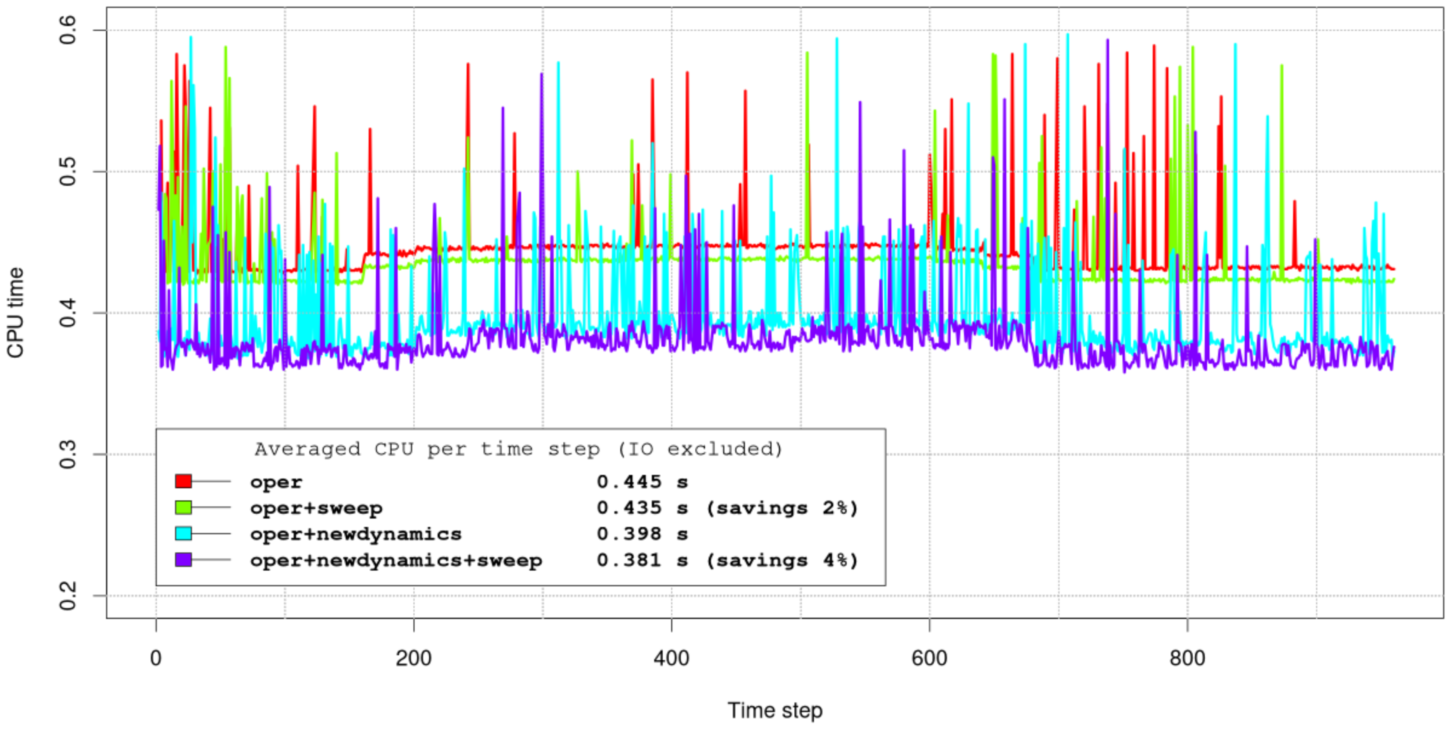
level 20

level 80



kinetic energy **vertical divergence**

Semi-Lagrangian horizontal diffusion



The CPU time savings depend on the platform and other choices in the model configuration: 2-4% for ALARO on NEC machine

Summary

- Sweep interpolations have been introduced in ACCORD LAM (based on CY48T3 and the implementation in IFS) recently.
- The method was made compatible with SLHD.
- The compatibility with COMAD is needed as well.
- The general performance shows similar accuracy as SL advection scheme with the cubic Lagrange interpolation with the possibility to control damping by the horizontal Laplacian smoother.
- The cost of sweep interpolations is reduced compared to standard method.

C-LAEF configuration at 1km - a winter case

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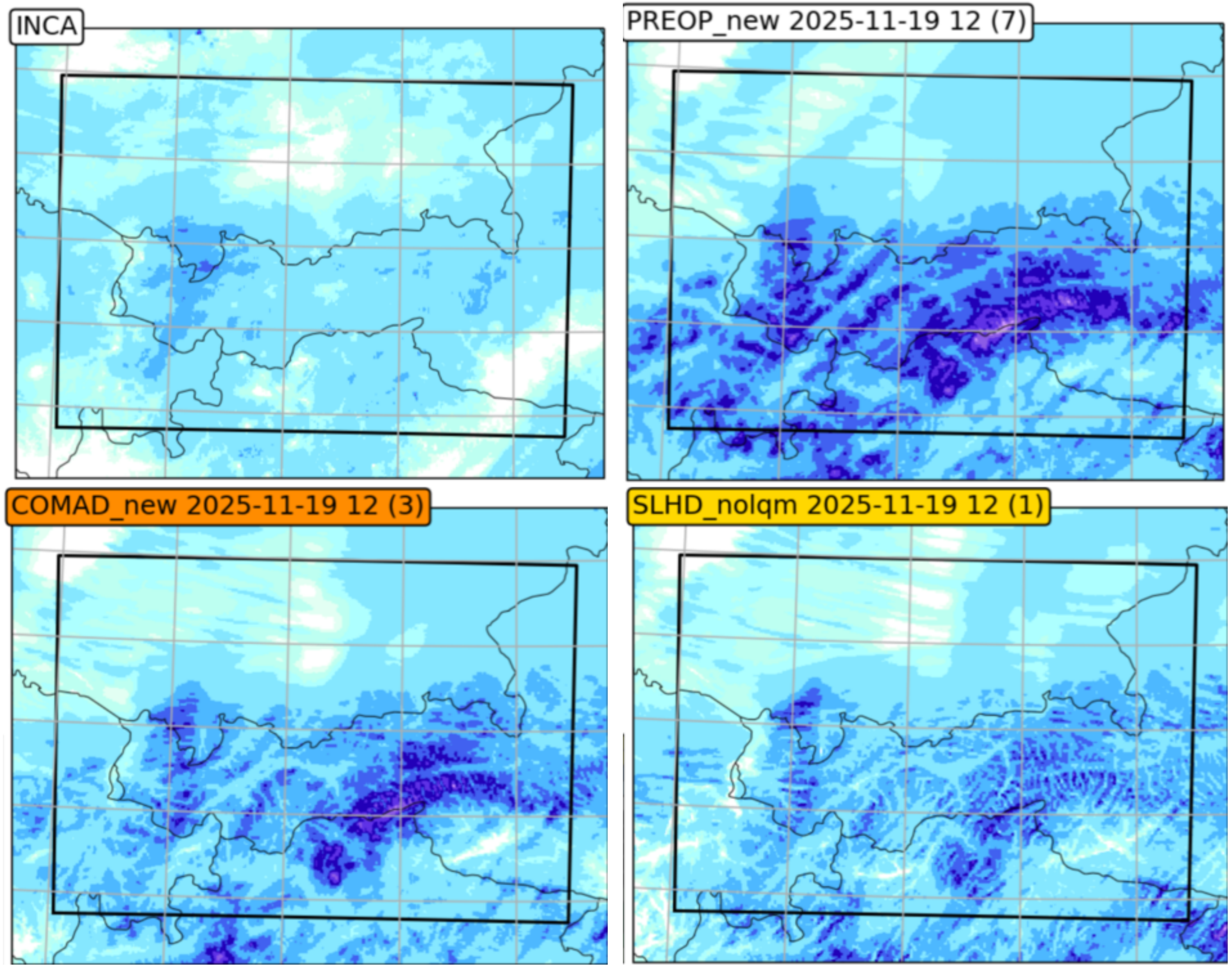
INCA
REF



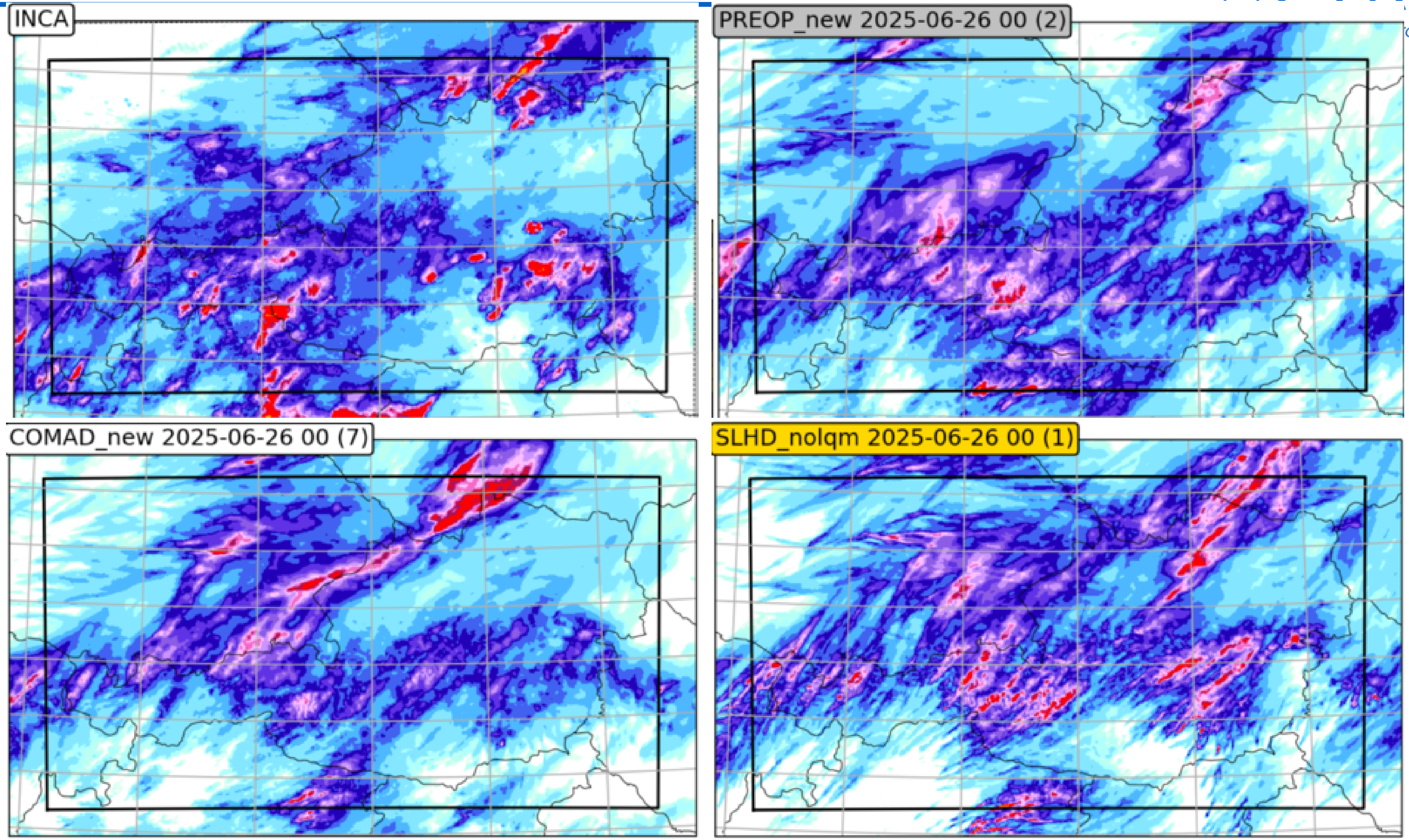
FIRST
TEST

SP.DIF.
+COMAD

SLHD

(courtessy of Clemens Wastl)

C-LAEF configuration at 1km - a summer case



(courtessy of Clemens Wastl)

Thank you for your attention!