

Regional Cooperation for  
Limited Area Modeling in Central Europe



A Consortium for CONvection-scale modelling  
Research and Development

## Dynamics for ACCORD forecast higher numerical consistency, stability and accuracy

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- ❑ Helmholtz elimination up to horizontal divergence
- ❑ New vertical divergence formulation
- ❑ New bottom boundary condition for vertical velocity
- ❑ Consistent inclusion of moisture in vertical motion variables
- ❑ Diagnostic definition of the orographic X-term in vertical divergence
- ❑ Blended approach for fields initialization

Previous solution: **LSI\_NHEE=F (default)**

- ❑ elimination up to vertical divergence  $\hat{d}$  in NHEE
- ❑ the vertical operators have to satisfy the necessary constraint
- ❑ space discretized solution does not correspond exactly the space continuous one ( $\mathbb{T} \neq \mathbb{I}$ )

$$[\mathbb{I} - \delta t^2 \nabla^2 \mathbb{B}_D^*] \hat{d} = d^{**}$$

$$\mathbb{B}_d^* = \frac{1}{1 - \kappa} \mathbf{H}_v^{*-1} (RT^* + \kappa g^2 \delta t^2 \mathbb{T})$$

Proposed solution: **LSI\_NHEE=T**

- ❑ elimination up to horizontal divergence  $D$  in NHEE
- ❑ no constraint
- ❑ similar to HPE elimination
- ❑ B-matrix has hydrostatic and non-hydrostatic part (allows for blended NH-HY approach)
- ❑ more suitable for VFE discretization

$$[\mathbb{I} - \delta t^2 \nabla^2 \mathbb{B}_D^*] D = D^{**}$$

$$\mathbb{B}_D^* = \underbrace{RT^* [\kappa \mathbf{G}^* \mathbf{S}^* + \mathbf{N}^*]}_{\text{hydrostatic}} + \underbrace{RT^* \frac{1}{1 - \kappa} \mathbf{G}_\kappa^* \mathbf{H}_v^{*-1} \mathbf{S}_\kappa^*}_{\text{nh increment}}$$

*(developed by Fabrice Voitus)*

**LGWADV=T (default from CY50T1)** allows for the usage of vertical divergence "  $d$  " in the linear model and vertical velocity  $w$  in the non-linear model with the transformations between them

$$\hat{d} = -\frac{p}{mRT} \frac{\partial gw}{\partial \eta}$$
$$w = \frac{1}{g} \int_{\eta}^1 \left( \frac{mRT}{p} \hat{d} \right) d\eta'$$

The advantages:  $d$  helps with stability of the implicit linear solver, while  $w$  with the accuracy of the non-linear residual.

Moreover, the "  $d$  " variable can be modified three times

$$d = \hat{d} + \mathbf{X}$$
$$\mathbf{X} = \delta_S \mathbf{X}^S + \delta_d \mathbf{X}^d + \delta_w \mathbf{X}^w$$

where

$\delta_S$  is mastered with **NVDVAR=4**

$\delta_d$  is mastered with **NVDVAR=5**

$\delta_w$  is mastered with **LBIGW=T**

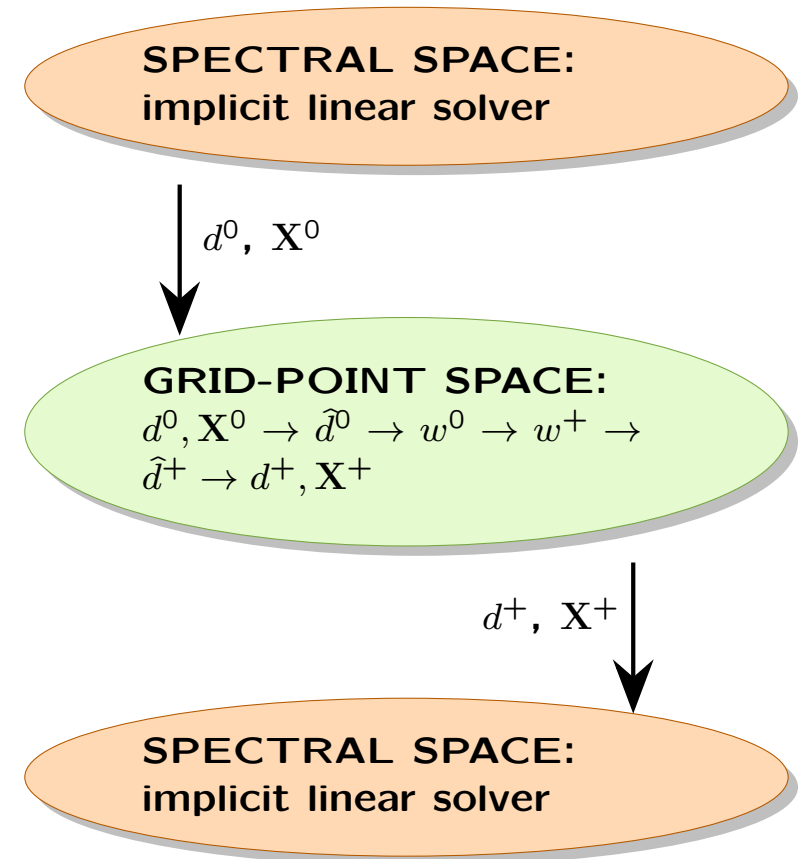
*(developed by Fabrice Voitus)*

At each time step or at each iteration of the ICI time scheme the following process is realized:

- the implicit part is realized in the SP space
- the vertical divergence  $d$  is transformed into the GP space, possibly together with the X-term (option `LSPNHX=TRUE`)
- the true vertical divergence  $\hat{d}$  is being calculated as  $\hat{d} = d - X$  and  $w$  is calculated from  $\hat{d}$
- this value is available as  $w^0$  in predictor and as  $w^{(k)}$  in the  $(k + 1)$ -th iteration (corrector)
- the new value  $w^{(k+1)}$  comes from

$$\frac{w_F^{(k+1)} - w_{O(k+1)}^0}{\Delta t} = \frac{RHS[w]_{O(k+1)}^0 + RHS[w]_F^{(k)}}{2}$$

- $w^{(k+1)}$  is transformed to  $\hat{d}^+$  and  $d^+$
- $d^+$  is transformed back into the SP space for implicit calculations for the next iteration or the next time step, possibly together with the newly calculated X-term (option `LSPNHX=TRUE`)



- The horizontal derivatives of the  $X$ -term are not needed in this case anywhere in the time marching scheme. (Not true for L3DTURB etc.)

## NVDVAR=3

$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa} (D + \hat{d} + \mathbf{X}^S)$$

$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa} (D + \hat{d} + \mathbf{X}^S + \mathbf{X}^d) + SD$$

$$\mathbf{X}^S = \frac{p}{mRT} \nabla \phi \frac{\partial \vec{v}}{\partial \eta}$$

$$\mathbf{X}^d = (1-\kappa) \left( \vec{v} \cdot \frac{\nabla \pi}{\pi} - \frac{1}{\pi} \int_0^\eta \vec{v} \cdot \nabla m \, d\eta' \right)$$

## NVDVAR=4

$$d_4 = \hat{d} + \mathbf{X}^S$$

$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa} (D + d_4)$$

$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa} (D + d_4 + \mathbf{X}^d) + SD$$

## NVDVAR=5

$$d_5 = d_4 + \mathbf{X}^d$$

$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa} (D + d_5 - \mathbf{X}^d)$$

$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa} (D + d_5) + SD$$

## In linear model

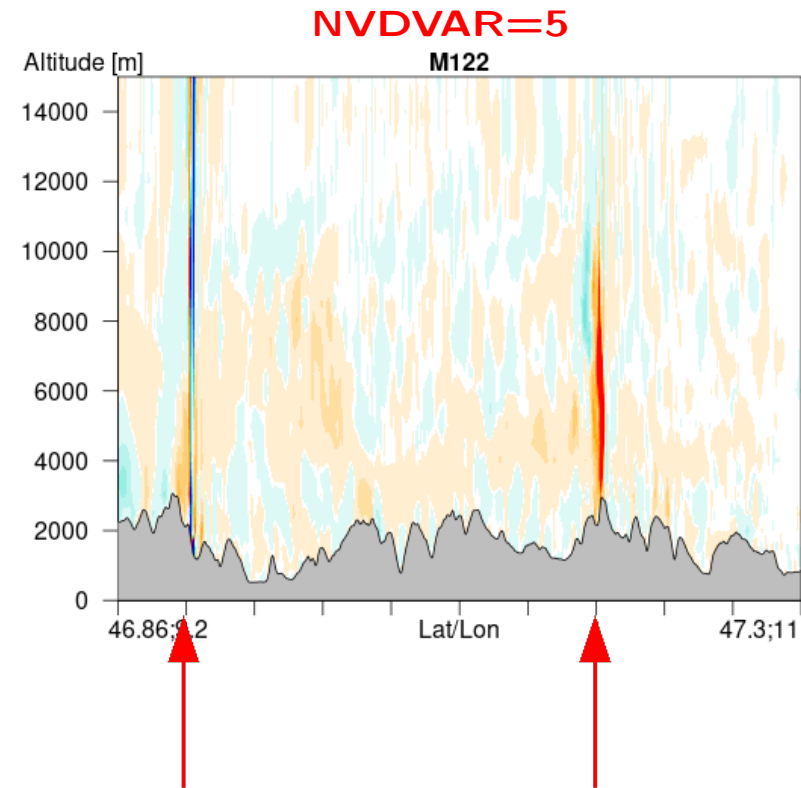
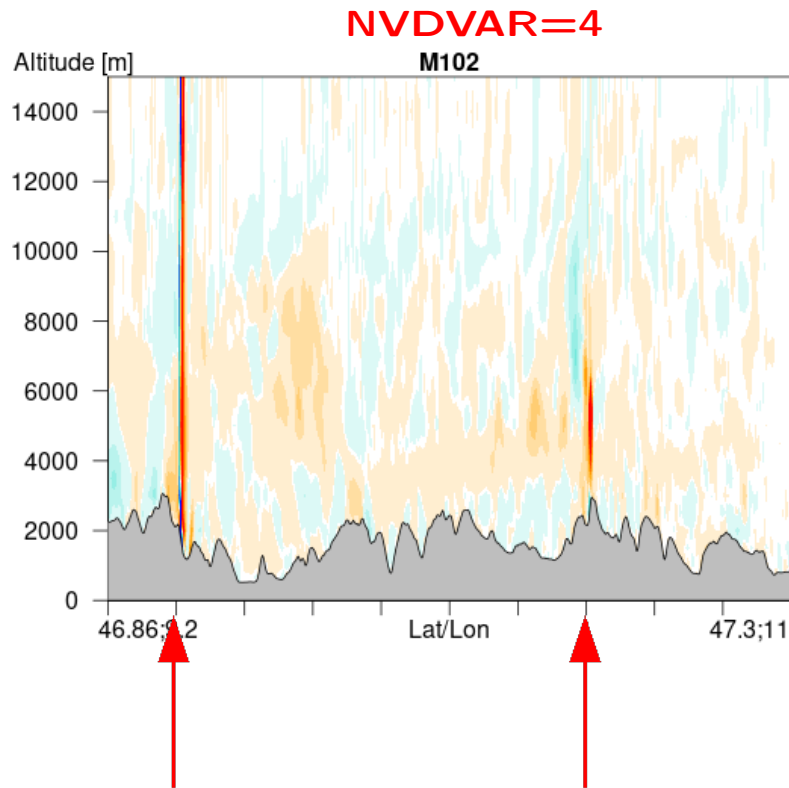
$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa} (D + d_i)$$

$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa} (D + d_i) + SD$$

The non-linear term  $\mathbf{X}^d$  is hidden inside the vertical divergence variable in  $\hat{q}$ -equation and temperature equation stays closer to the HPE solution

$$\frac{dT}{dt} = \kappa T \left( \frac{1}{1-\kappa} \mathbf{X}^d - SD \right)$$

(developed by Fabrice Voitus)



- more consistent
- the chimney like patterns still persistent

**LBIGW=.TRUE.**

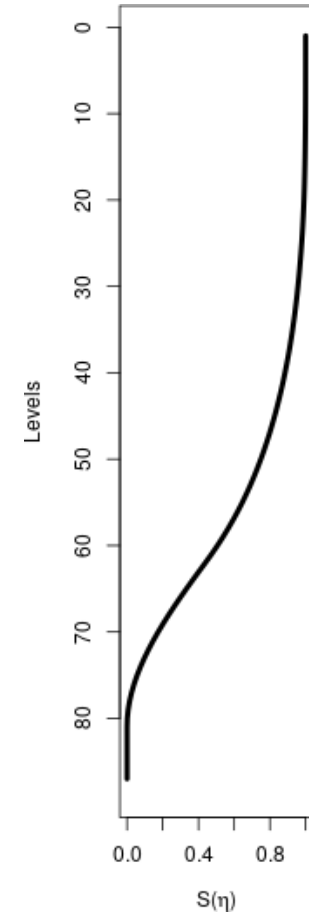
New vertical velocity  $gW = gw - S(\eta)\vec{V} \cdot \nabla\Phi_S$

where 
$$S(\eta) = \frac{B(\eta)\pi_{ref}}{A(\eta) + B(\eta)\pi_{ref}}$$

- $S = S(\eta)$  is a prescribed monotonic vertical function satisfying  $S(0) = 0$  at the top, and  $S(1) = 1$  at the bottom
- $S(\eta)\nabla\Phi_S$  fits  $\nabla\Phi$  for a stationary isothermal hydrostatic atmosphere
- $W$  behaves as  $w$  at the top and as  $\dot{\eta}$  at the bottom.
- Rigid BBC reads  $W_S = 0$ .
- It can be seen as a third X-term part added to vertical divergence definition

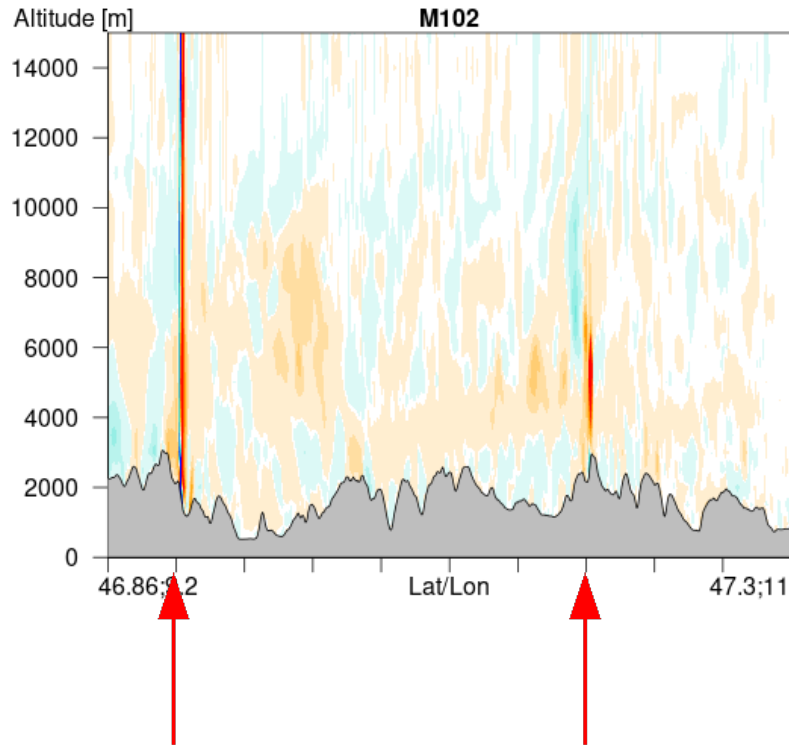
$$d_i^w = d_i + \mathbf{X}^w$$

$$\mathbf{X}^w = -\frac{p}{mRT} \frac{\partial}{\partial \eta} (gW - gw)$$

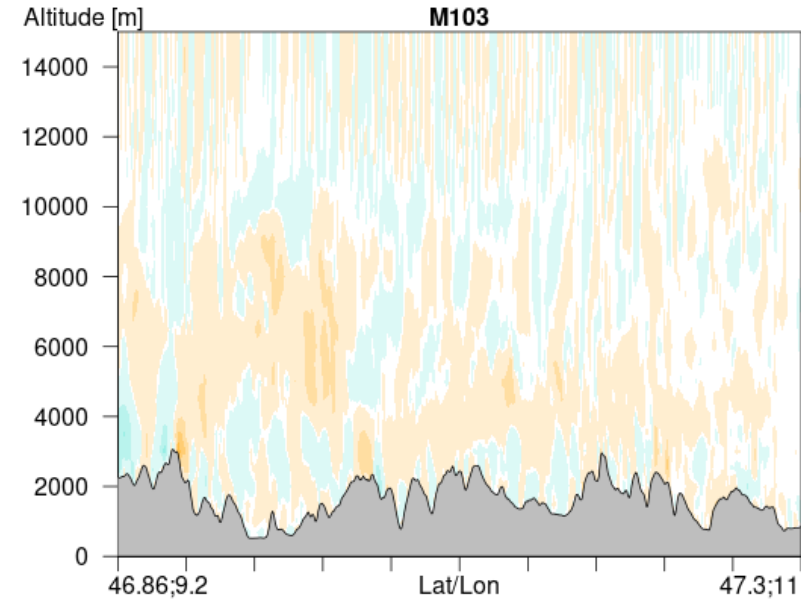


(developed by Fabrice Voitus)

**LBIGW=.FALSE.**



**LBIGW=.TRUE.**



- simple BBC
- the chimney like patterns disappear

## L\_RDRY\_VD=T (default)

The definition of the vertical divergence variable is made consistently everywhere in the code with the dry variant of the gaz constant  $R_d$ .

$$\hat{d} = -\frac{p}{m(\delta_v R_d + (1 - \delta_v)R)T} \frac{\partial g_w}{\partial \eta}$$

The definition of X-term is treated independently depending on the key L\_RDRY\_NHX.

## L\_RDRY\_NHX=T

The definition of the X-term is made consistently everywhere in the code with the dry variant of the gaz constant  $R_d$ .

$$X = \frac{p}{m(\delta_X R_d + (1 - \delta_X)R)T} \nabla \phi \frac{\partial \vec{v}}{\partial \eta}$$

The true 3D-divergence is always calculated with the moist  $\hat{d}$ .

*(developed by Fabrice Voitus)*

ND4SYS=1 and ND4SYS=2

The time evolution of  $\hat{d}$  is being solved by

$$\frac{d\hat{d}}{dt} = RHS[\hat{d}] + \frac{dX}{dt}$$

Then  $\frac{dX}{dt}$  is discretized in the following way:

$$\frac{dX}{dt} = \frac{X_F^0 - X_{O(k+1)}^0}{\Delta t}$$

for ND4SYS=1 oscillating in time

$$\frac{dX}{dt} = \underbrace{\frac{X_F^{(last)} - X_F^0}{\Delta t}}_{\text{last iteration only}} + \frac{X_F^0 - X_{O(k+1)}^0}{\Delta t}$$

for ND4SYS=2 less oscillating

last iteration only

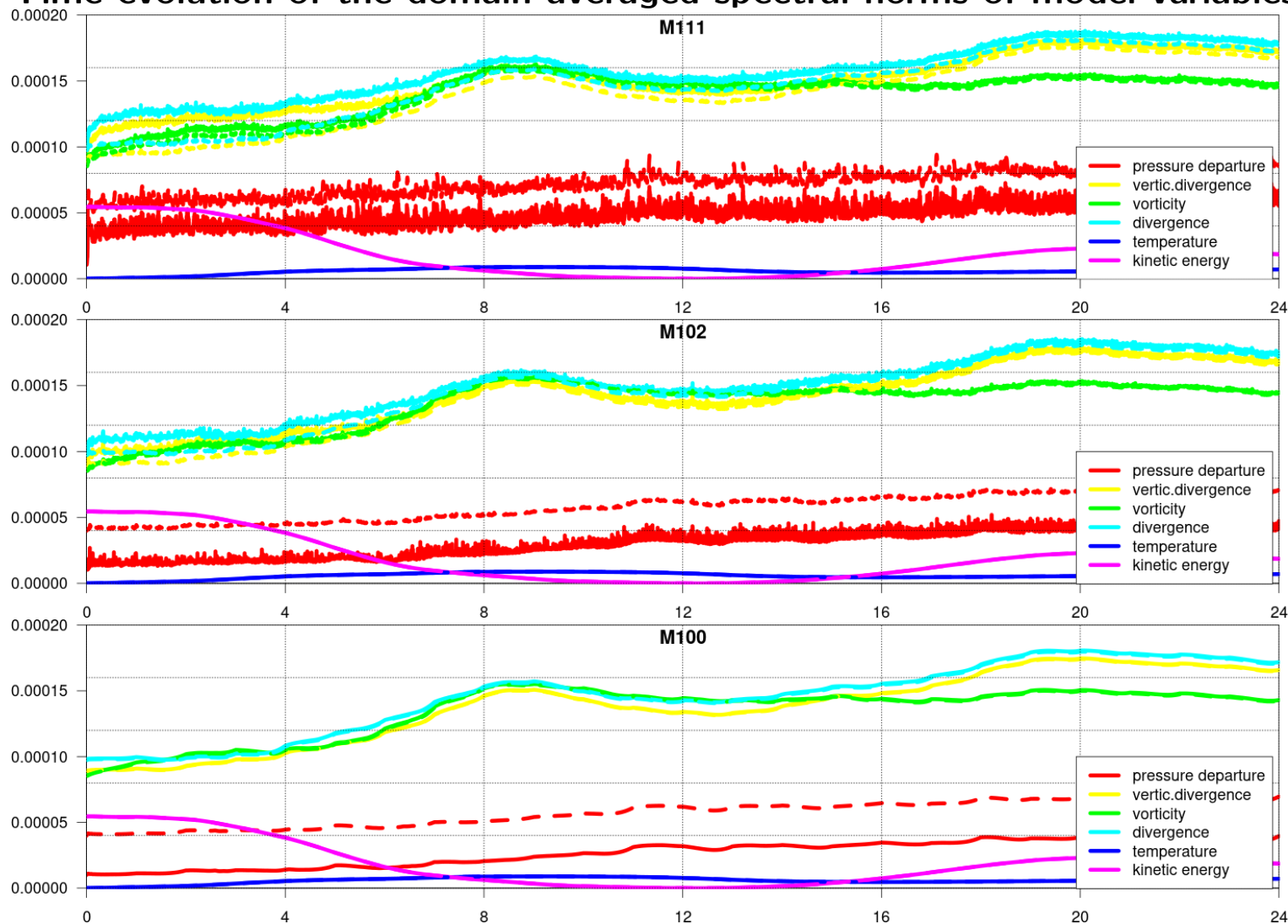
ND4SYS=0

$$\hat{d} = \Delta t RHS[\hat{d}] + X$$

The X-term may be calculated at the beginning and at the end of the grid-point calculations. The transformation of the X-term to and from spectral space may be avoided resulting in the reduced usage of the CPU time ( $\approx 6 - 8\%$  with `LSPNHX=.TRUE.`).

# Diagnostic treatment of the X-term

## Time evolution of the domain averaged spectral norms of model variables

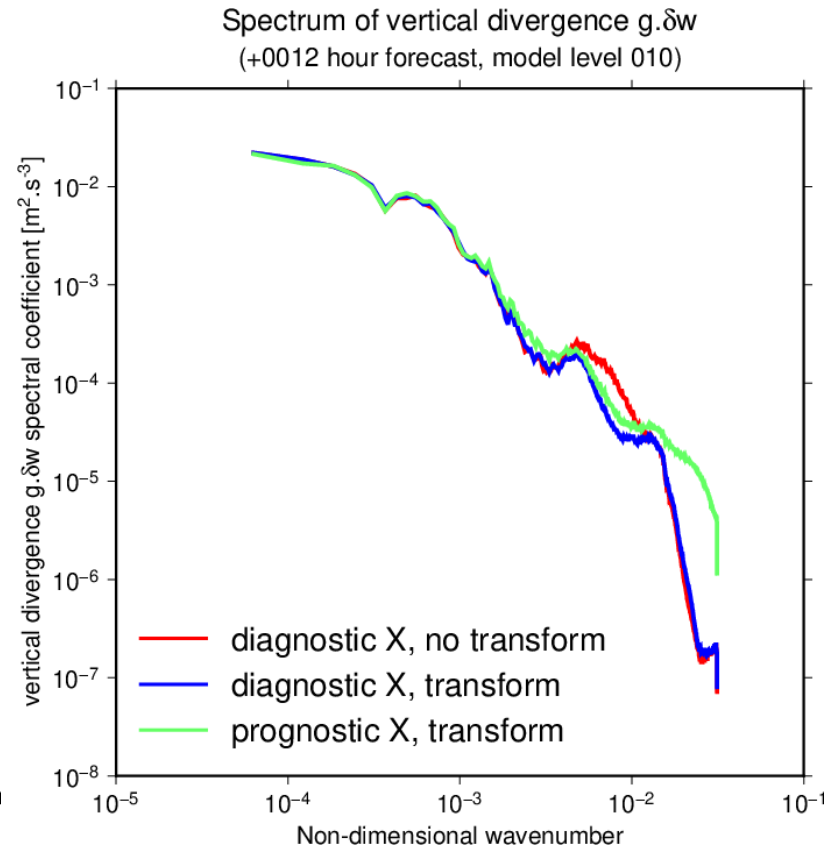
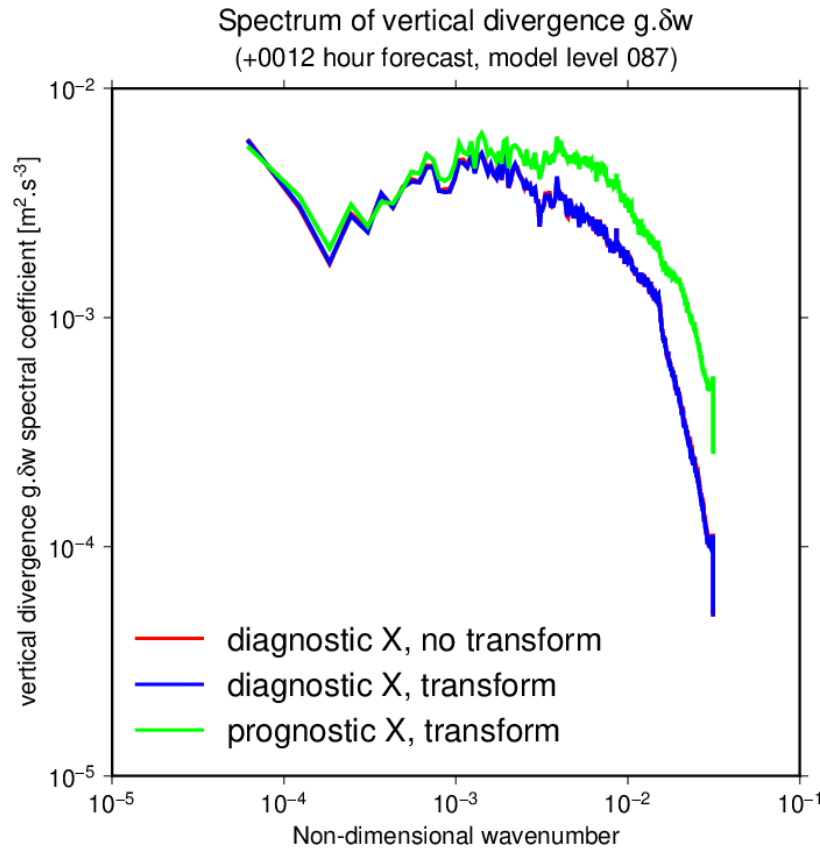


ND4SYS=1

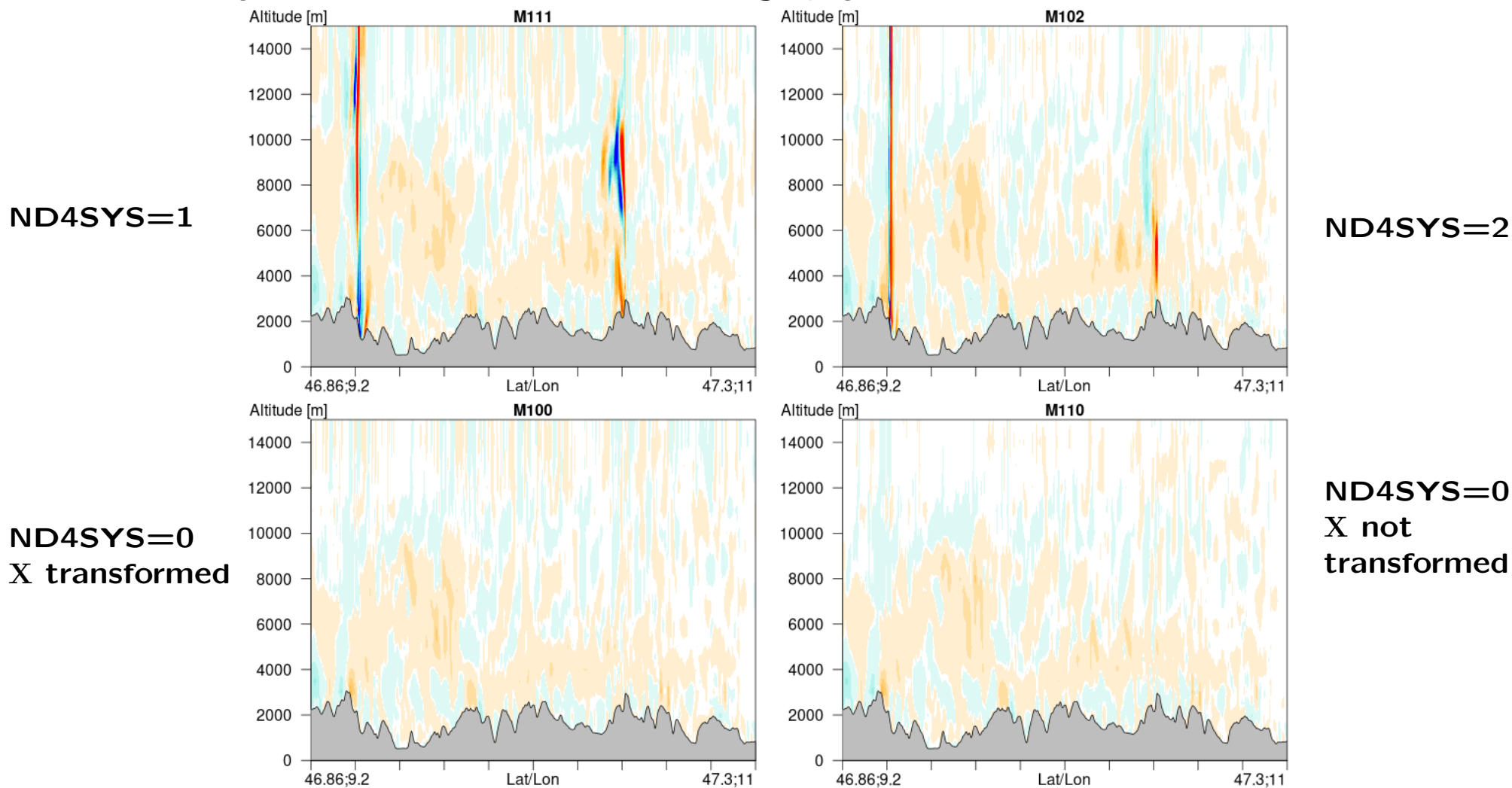
ND4SYS=2  
prognostic X

ND4SYS=0  
diagnostic X

## Vertical divergence spectra for the model levels 87 and 10



## Vertical velocity vertical cross section above orography



We use one control parameter  $\delta$ ,  $0 \leq \delta \leq 1$ .

$$\begin{aligned} \frac{dT}{dt} &= \kappa T \frac{\dot{\pi}}{\pi} - \delta \frac{\kappa T}{(1 - \kappa)} (d_5 + \mathbf{S}_\kappa D) \\ \frac{d\vec{v}}{dt} &= -RT \frac{\nabla \pi}{\pi} - \nabla \phi - \delta \left( RT \nabla \hat{q} + \frac{1}{m} \frac{\partial(p - \pi)}{\partial \eta} \nabla \phi \right) \\ \frac{dgw}{dt} &= g^2 \frac{\delta}{m} \frac{\partial(p - \pi)}{\partial \eta} \\ \frac{d\hat{q}}{dt} &= -\frac{\delta}{(1 - \kappa)} (d_5 + \mathbf{S}_\kappa D) \\ \frac{\partial q_s}{\partial t} &= -\frac{1}{\pi_s} \int_0^1 D \, d\eta \\ D &= \nabla \cdot m\vec{v}, \end{aligned}$$

where

$$\begin{aligned} \frac{\dot{\pi}}{\pi} &= \frac{1}{1 - \kappa} \mathbf{X}^d - \mathbf{S}D \\ \phi &= \phi_s + \int_\eta^1 \frac{mRT}{\pi} \, d\eta' - \delta \int_\eta^1 \frac{mRT}{p} \left( \frac{p - \pi}{\pi} \right) \, d\eta'. \end{aligned}$$

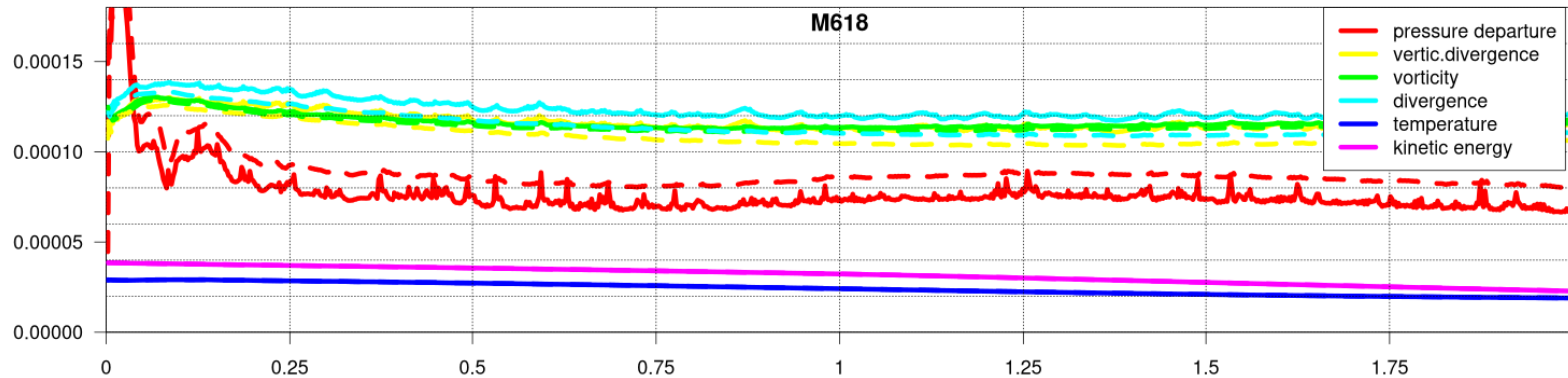
HPE system  $\sim \delta = 0$

EE system  $\sim \delta = 1$

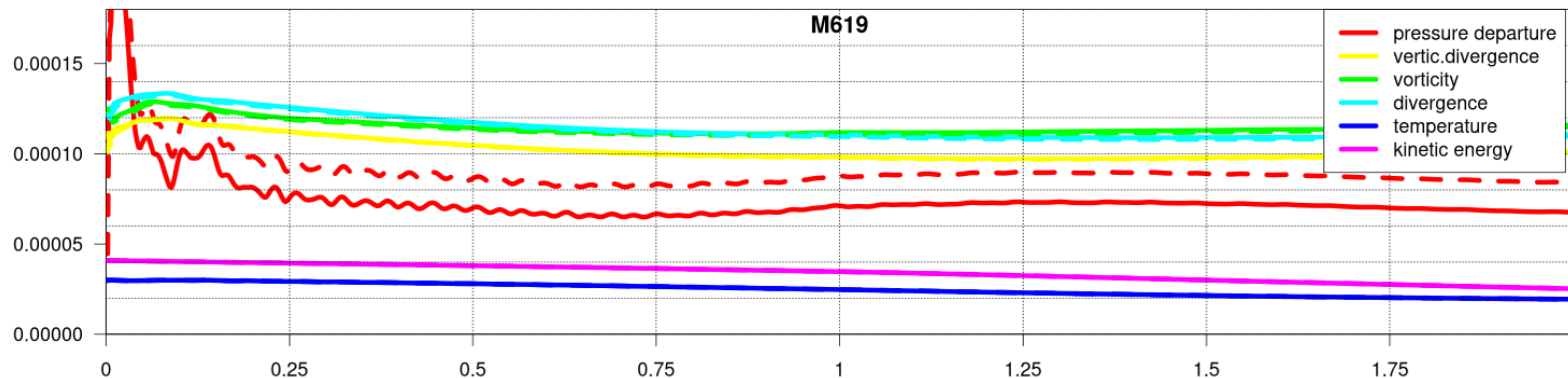
a blended system is available with  $0 < \delta < 1$

- filtering fast moving waves without meteorological relevance
- may guarantee higher numerical stability

## Coupling from global model, initial conditions from global model



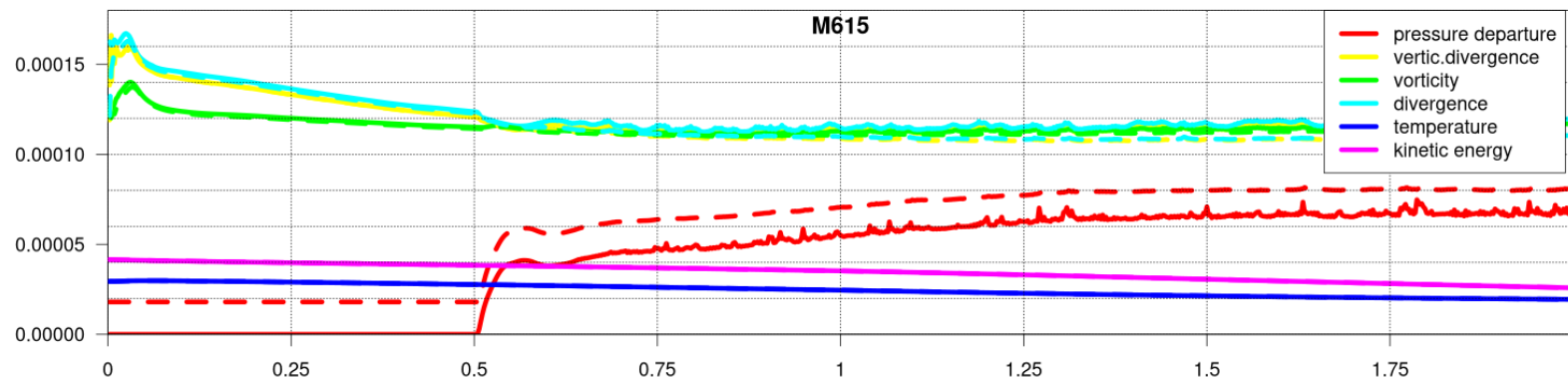
ND4SYS=2  
NVDVAR=4  
LBIGW=F



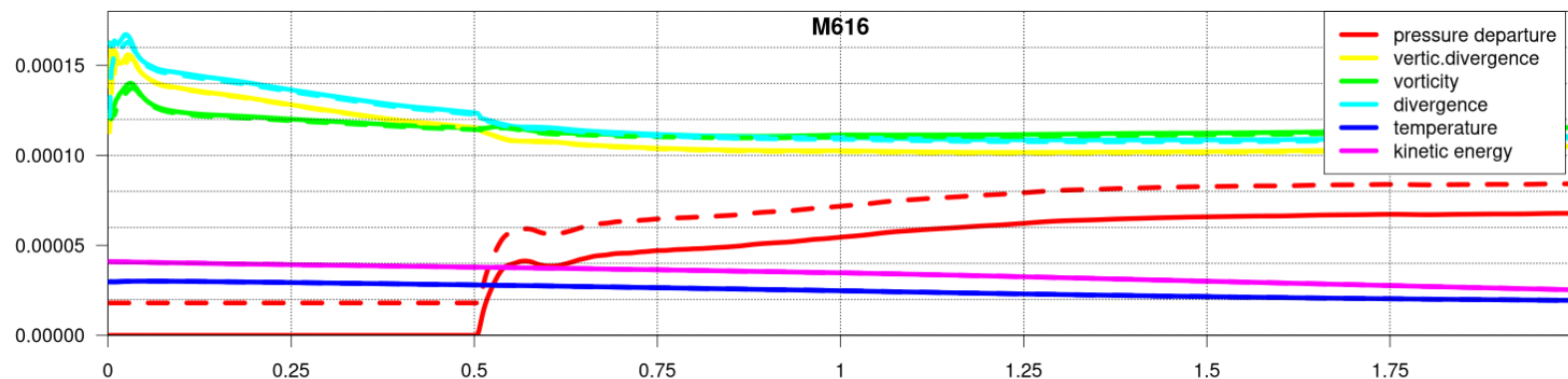
ND4SYS=0  
NVDVAR=5  
LBIGW=T

# Blended approach for fields initialization

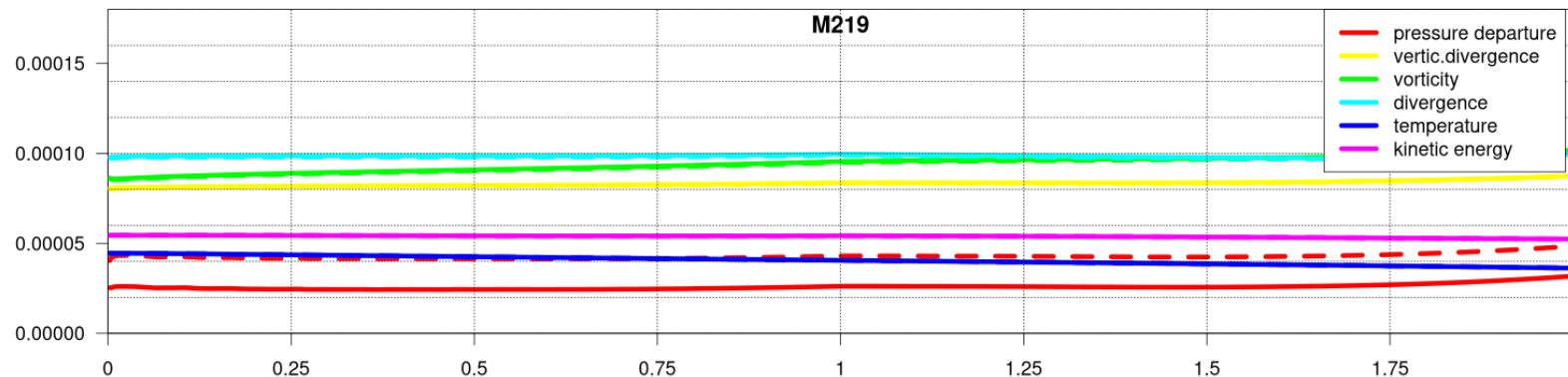
HPE, blended approach 30min - 2hours, NH



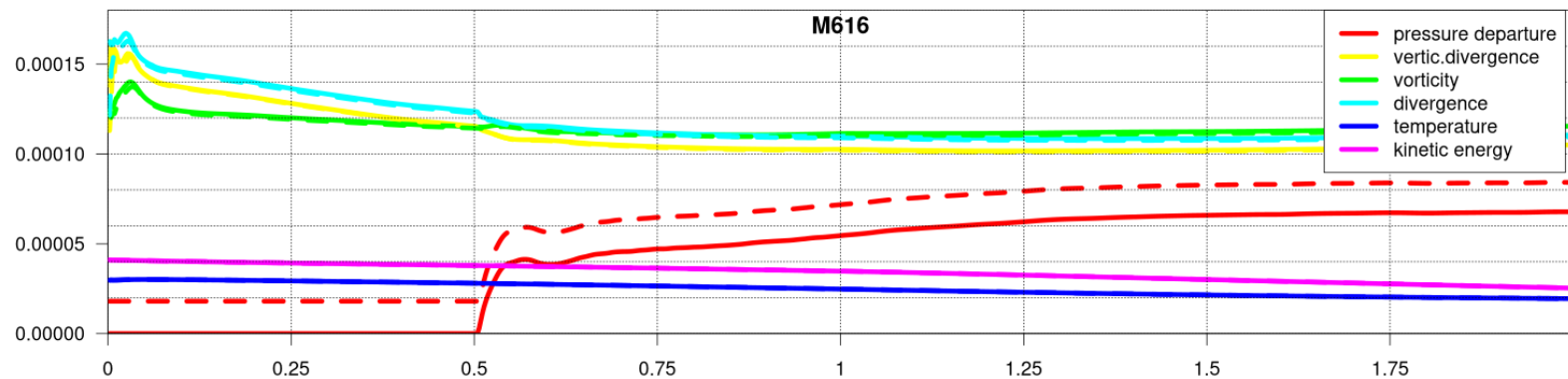
ND4SYS=2  
NVDVAR=4  
LBIGW=F



ND4SYS=0  
NVDVAR=5  
LBIGW=T

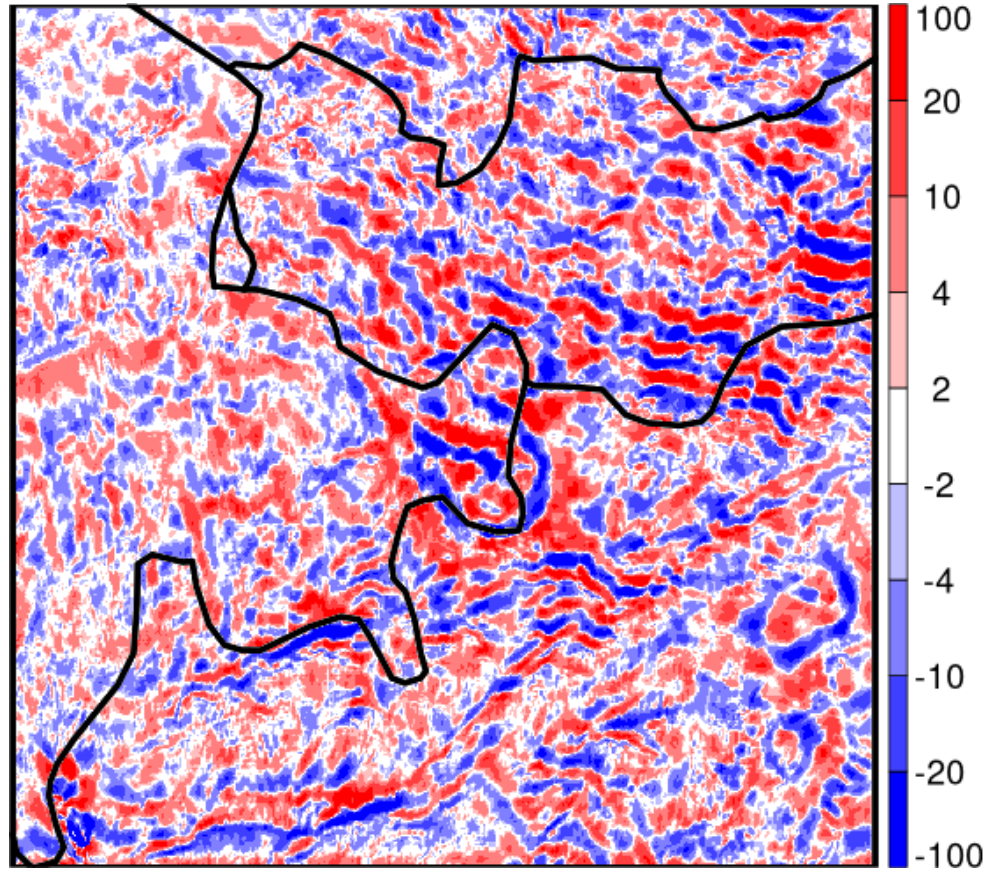


coupling from  
HRES LAM  
and DFI

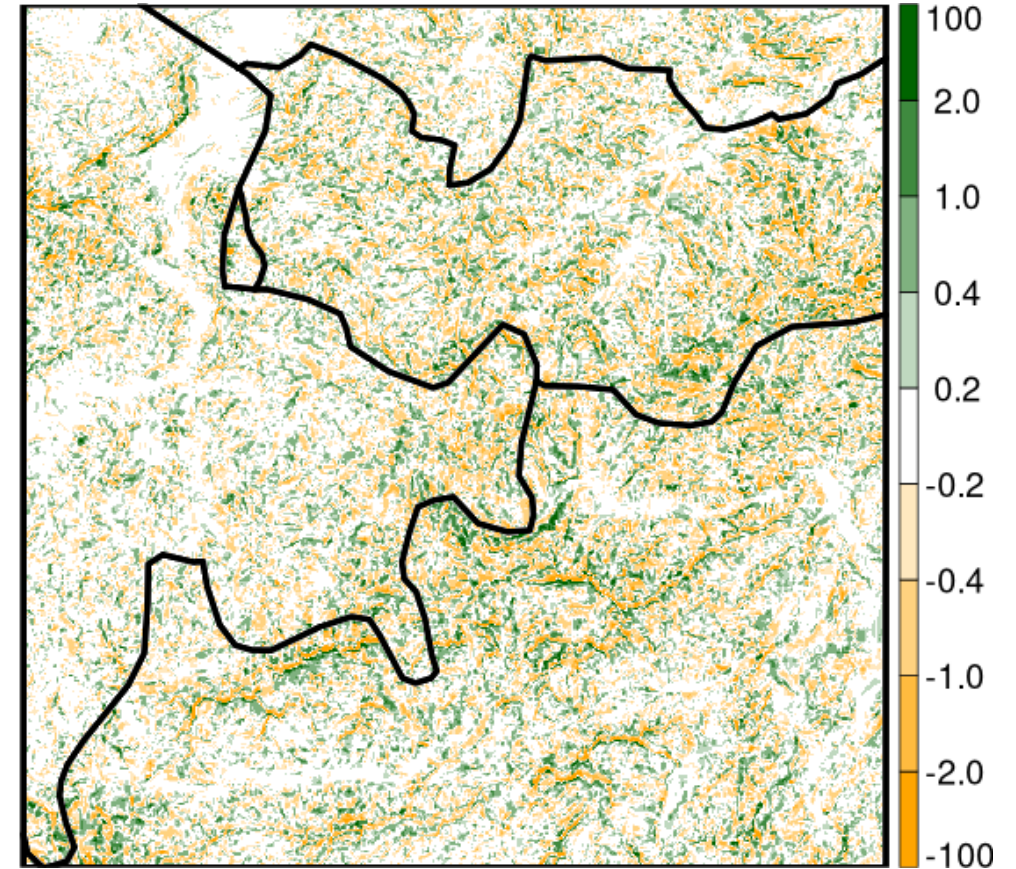


ND4SYS=0  
NVDVAR=5  
LBIGW=T

## Difference between NH and blended approach

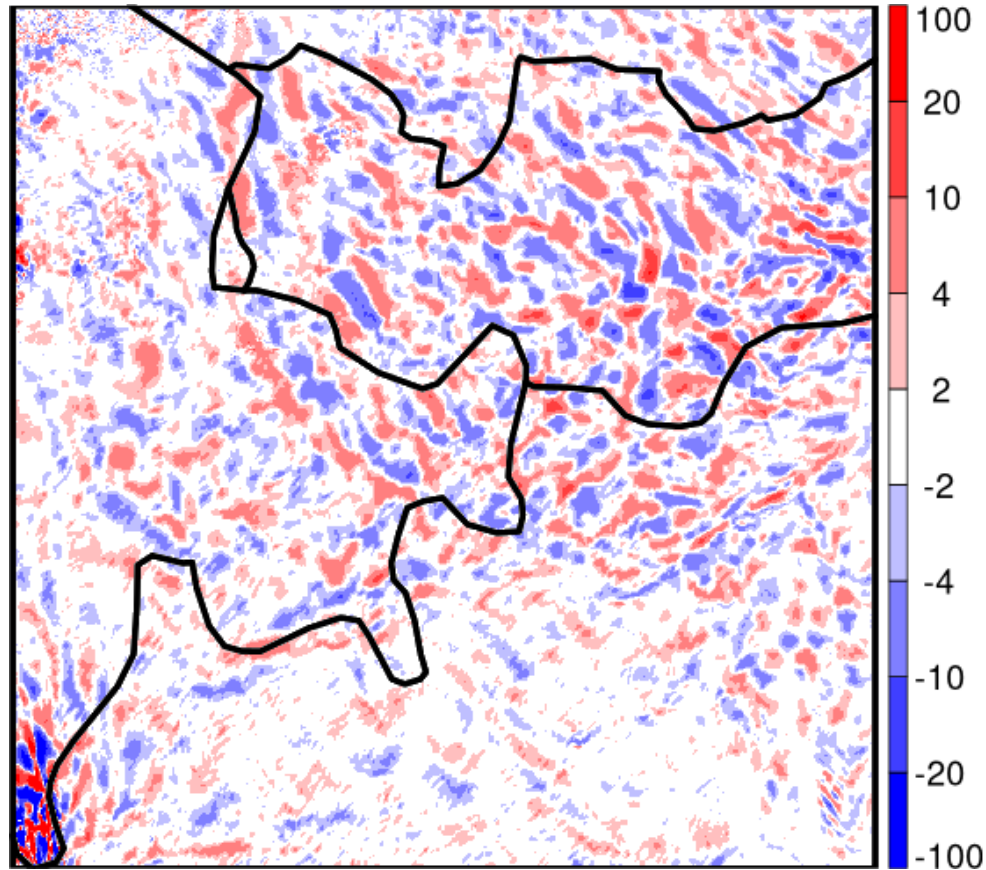


+ 30 minuts: pressure departure

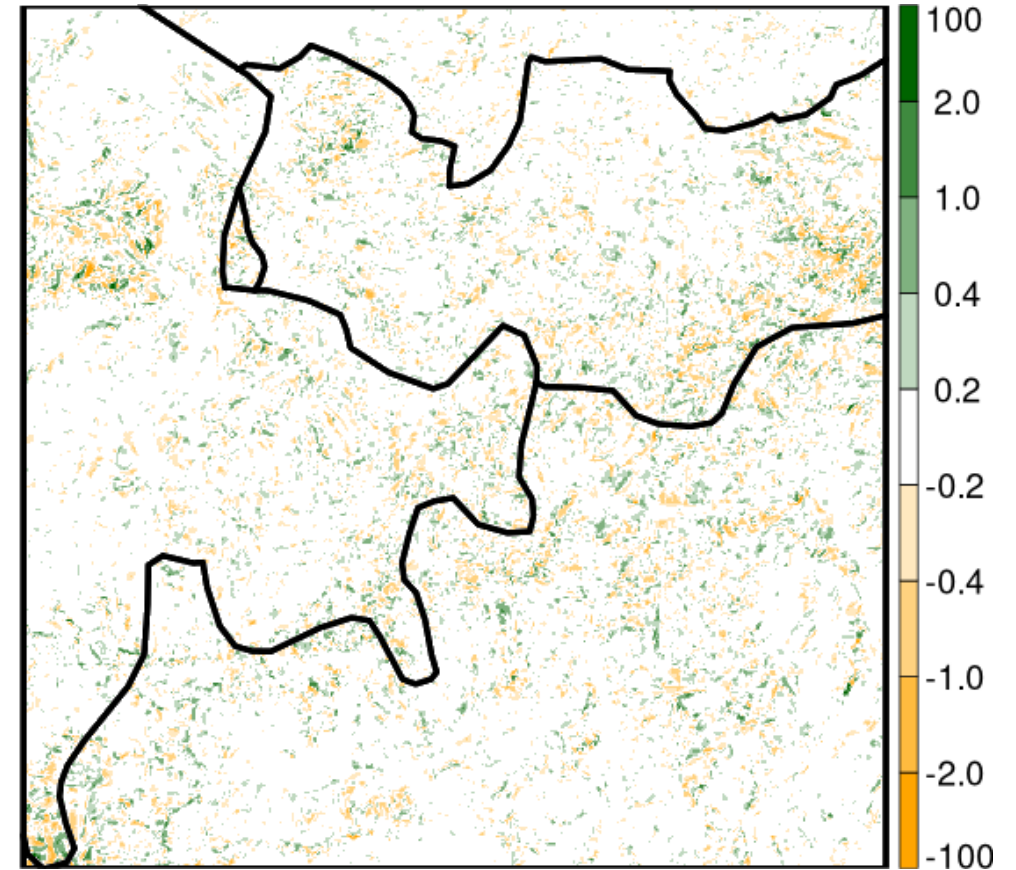


vertical divergence

## Difference between NH and blended approach

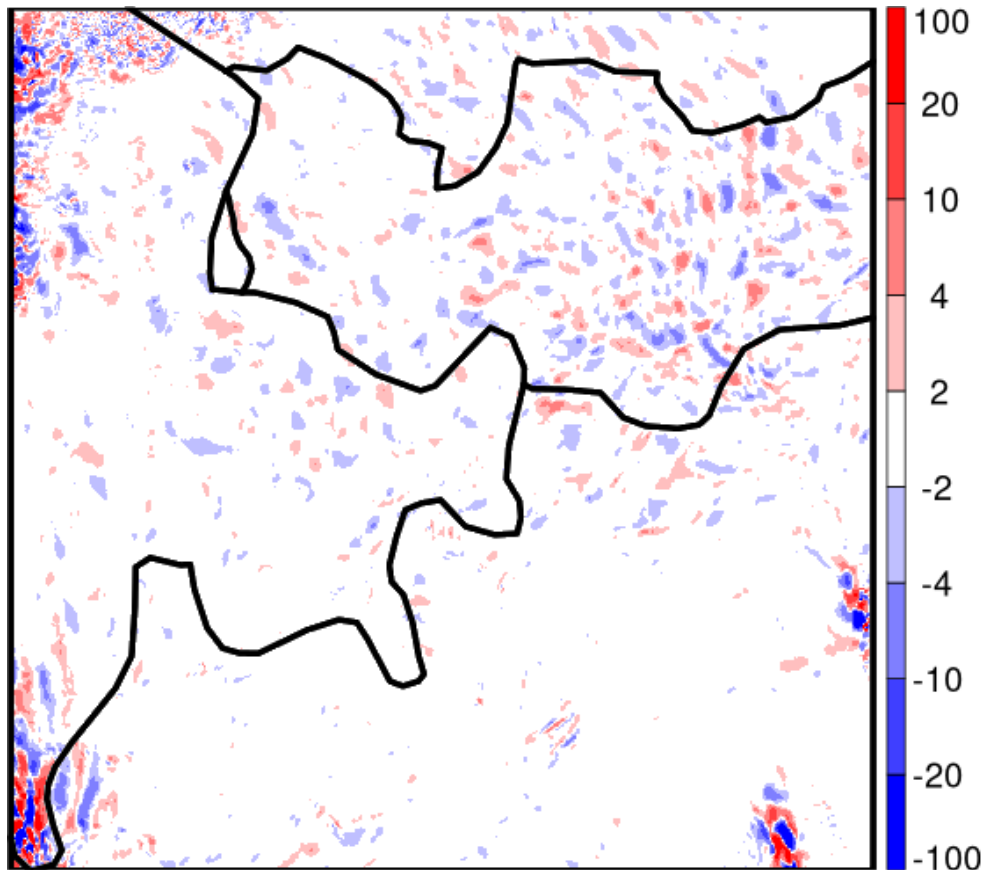


+ 1 hours: pressure departure

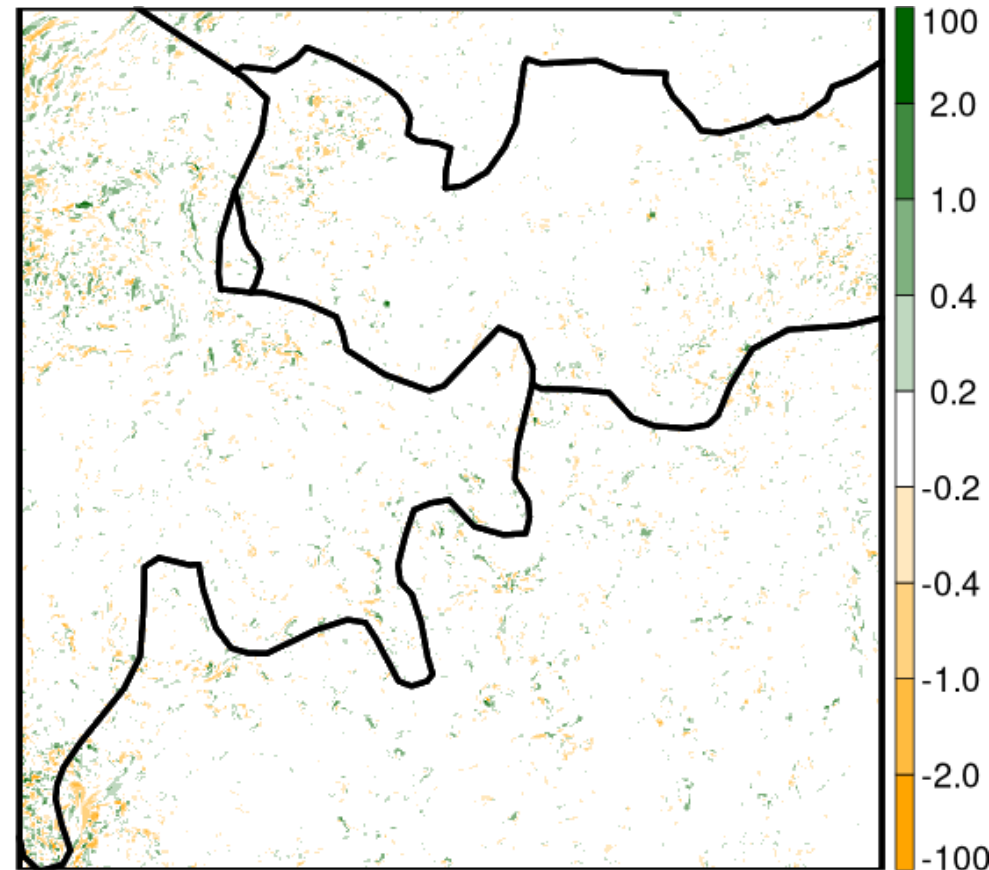


vertical divergence

## Difference between NH and blended approach



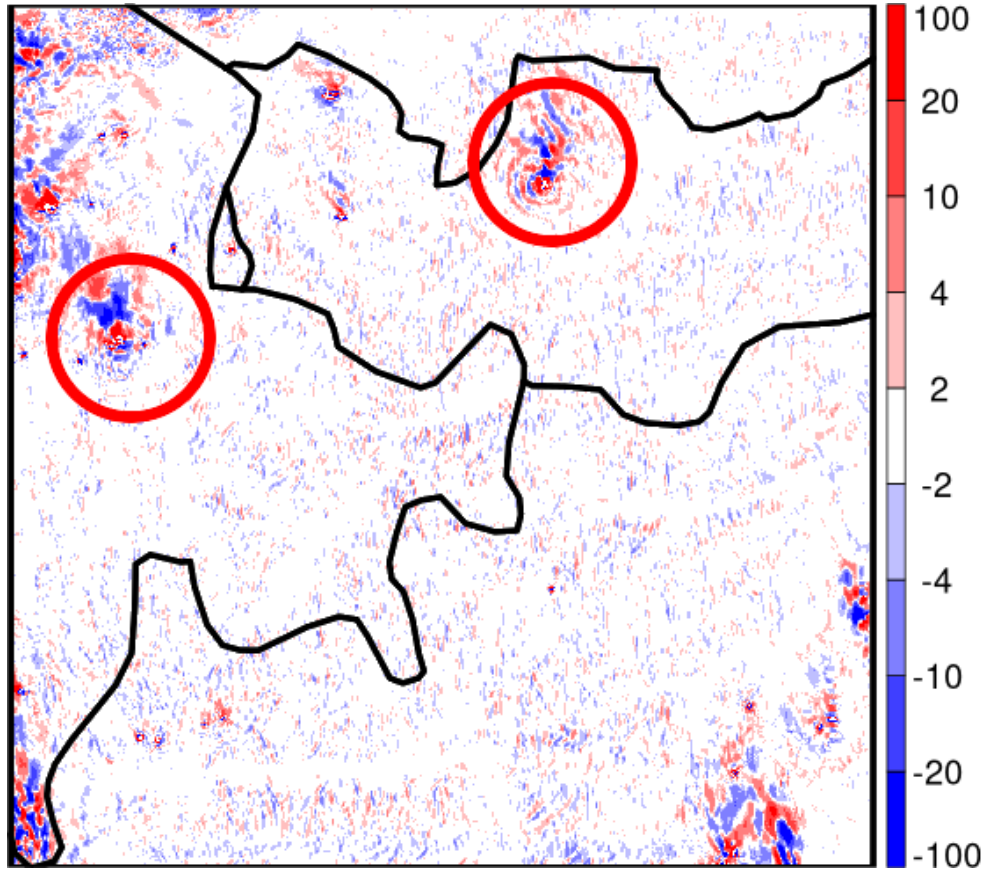
+ 2 hours: pressure departure



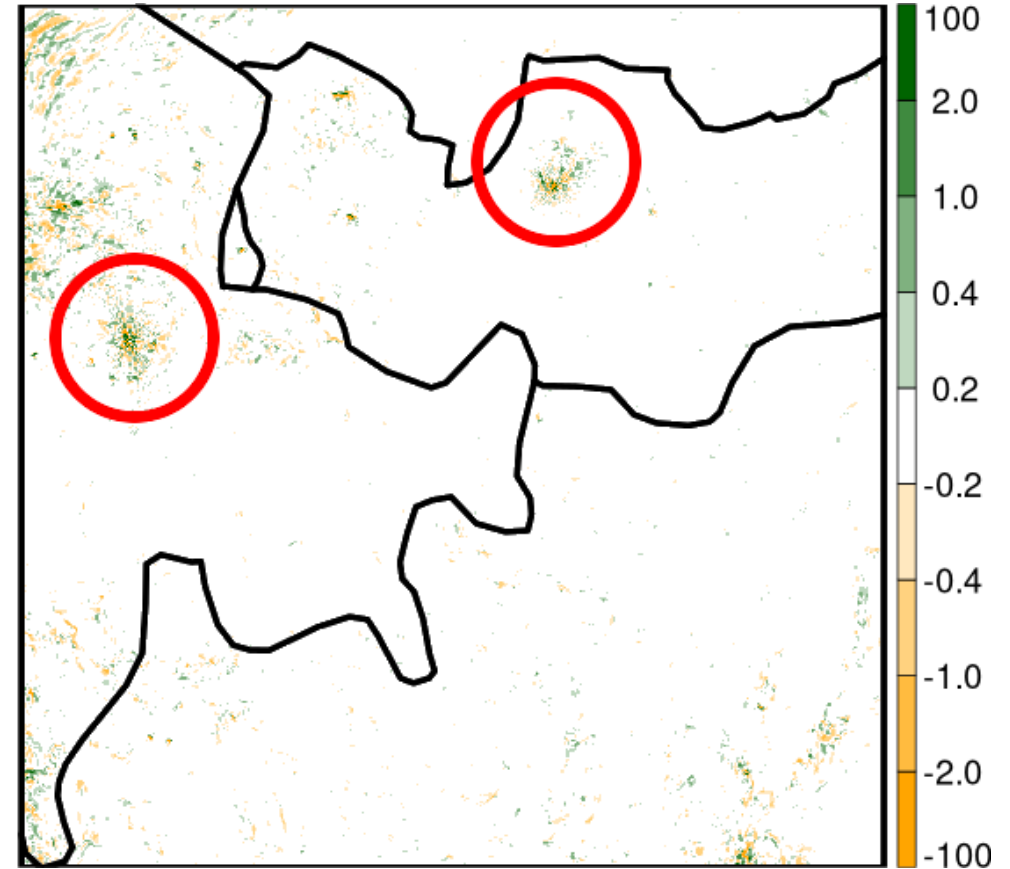
vertical divergence

# Blended approach for fields initialization

Difference between NH with "old dynamics" and NH with "new dynamics"



+ 2 hours: pressure departure



vertical divergence

New dynamic options available in CY48T3\_deode:

**&NAMDYNA**

L\_RDRY\_NHX=.T.,

LBIGW=.T.,

LSI\_NHEE=.T.,

LSPNHX=.F.,

ND4SYS=0,

NVDVAR=5,

/ ...

Will be available in CY49T2\_deode and CY50T1 soon 😊.

**Köszönöm a figyelmüket!**

**Thank you for your attention!**