On the design of a more robust Constant-Coefficient Predictor-Corrector scheme for ACCORD Models

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Plan

Constant-coefficient Iterative-Centered-Implicit (ICI) scheme

$$\frac{\partial \mathcal{X}}{\partial t} = \mathcal{M}(\mathcal{X}) = \underbrace{(\mathcal{M} - \mathcal{L}^*)(\mathcal{X})}_{\textit{NL-residual}} + \underbrace{\mathcal{L}^*.\mathcal{X}}_{\textit{Linear}}$$

- Iterative centred implicit resolution, for $i \in [0, N_{\text{siter}} - 1]$:

$$\frac{\mathcal{X}^{+(i+1)} - \mathcal{X}^0}{\delta t} = \frac{(\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^{+(i)}\right) + (\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^0\right)}{2} + \frac{\mathcal{L}^*.\mathcal{X}^{+(i+1)} + \mathcal{L}^*.\mathcal{X}^0}{2},$$

- **Quasi-Newton-Raphson approach** : $[I - (\delta t/2)\mathcal{L}^*]$ plays as a preconditioner $\Rightarrow \mathcal{L}^*$ is typically chosen as a linear counterpart of $\mathcal{M}$ around a reference-state $\mathcal{X}^*$.
- **Constant-coefficient assumption :** The coefficients of $\mathcal{L}$ are taken constant in time and along horizontal directions $\Rightarrow$ Make possible the inversion of $[I - (\delta t/2)\mathcal{L}^*]$ in spectral space.

Constant-coefficient Iterative-Centered-Implicit (ICI)

Major drawbacks of constant-coefficient assumption :

- Stability strongly depend on the magnitudes of the non-linear residual terms ($\mathcal{M} - \mathcal{L}^*$). Such issues even are more significant in fully-compressible (EE) mass-based models [Bénard *et al.* (2003,2004,2005)].

1. **Thermal NL residuals** due to discrepancy between the actual temperature and the constant temperature used in the SI linear system $\Rightarrow$ Instability issue in presence too cold temperature in the top of the atmosphere.
2. **Orographic NL residuals** due to the presence in of horizontally-varying terrain-following metric terms involving the orography slope that are not taken into account in the SI linear model $\Rightarrow$ Steep slopes create stability issue.
3. **Baric NL residuals** (specific to mass-based system) due to the discrepancy between actual hydrostatic surface pressure and the constant hydrostatic pressure entering in the definition of the vertical operartors of the SI linear system $\Rightarrow$ Stability issue in presence of high orography plateau (e.g, Tibetan plateau region).

Current Constant-Coefficient SI linear of mass-based EE system : $\mathcal{L}^*$

$$\frac{\partial D}{\partial t} = -R_d \nabla^2 \left[\left(\mathcal{G}^*(T) + T_r^* q_s \right) + T_r^* \left(\hat{q} - \mathcal{G}^*(\hat{q}) \right) \right]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{R_d T_a^*} \mathcal{L}^* (\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{R_d T_r^*}{C_{vd}} [D + d]$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_{pd}}{C_{vd}} [D - (1-\kappa)\mathcal{S}^*(D) + d]$$

$$\frac{\partial \mathrm{l}_s}{\partial t} = -\mathcal{N}^*(D)$$

Current Constant SI parameters :

- **SITR** : Warm constant SI temperature T_r^* $\Rightarrow$ impact on Thermal NL residuals related to gravity modes and horizontal elastic term (Lamd's mode).
- **SITRA** : Cold constant acoustic SI temperature T_a^* $\Rightarrow$ impact on Thermal NL residuals due to vertical elastic term.
- **SIPR**: Constant SI hydrostatic surface pressure π_{sr}^* $\Rightarrow$ Impact on Baric NL residuals.

- (**SITR**, **SITRA**, **SIPR**) are set once for all at the beginning of the Model integration. $\Rightarrow$ Need to be **tuned** over long period of tests trying to prevent that the Model blows up or exhibits noisy solution **for each domain**.

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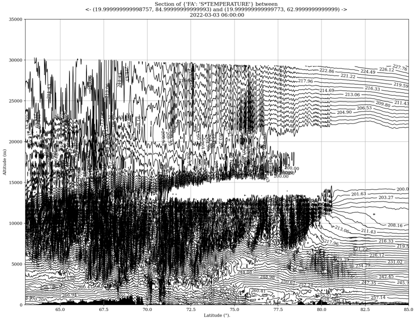
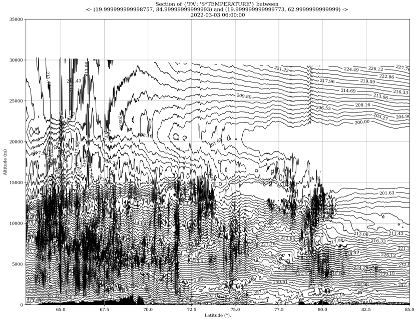
Svalbard problematic run with current PC scheme (NSITER=1)

SITR$=350\,K$ and **SITRA**$=100,\,K$



SITR$=300\,K$ and **SITRA**$=140,\,K$



- *In Svalbard case,* $T_{max}(t)$ *never exceeds* $290K$ *and* $\min[T^{\ell}_{max}(t)]$ *reactches very cold value around* $190K \Rightarrow$ *Current value* ***SITR*** $= 350K$ *exaggerates too much the thermal NL residuals* $\Rightarrow$ *generate noisy solutions.* can not damp.

Design of the vertically-dependent updated SI parameters

Some useful definitions :

- Max/Min at each vertical levels $\ell \in [1, L]$:

$$T^{\ell}_{max}(t) = \max_{x \in \mathcal{D}_h} T(x, \ell, t)$$

$$T^{\ell}_{min}(t) = \min_{x \in \mathcal{D}_h} T(x, \ell, t)$$

- Global Max/Min over the domain :

$$T_{max}(t) = \max_{\ell \in [1, L]} T^{\ell}_{max}(t)$$

$$T_{min}(t) = \min_{\ell \in [1, L]} T^{\ell}_{min}(t)$$

Proposed definitions :

- Reduce Thermal NL residual by defining vertically and time-dependent SI parameters from the grid-point norms min/max of temperature at each level as :

$$T_r^{\ell}(t) = T_{max}^{\ell}(t) + \min\left[\left(T_r^* - T_{max}(t)\right), \beta_r\left(T_{max}^{\ell}(t) - T_{min}^{\ell}(t)\right)\right],$$

$$T_a^{\ell}(t) = \frac{T_{min}^{\ell}(t)}{2\beta_a + 1}, \qquad \text{for} \ell \in [1, L],$$

$$T_r^{s}(t) = T_{max}(t)$$

- where β_r and β_a are control parameters lying in $[0, 1]$.

Design of the vertically-dependent updated SI parameters

$$\frac{\partial D}{\partial t} = -R_d \nabla^2 \left[\left(\mathcal{G}^* (T) + T_r^s q_s \right) + \left(T_r^\ell \hat{q} - \mathcal{G}^* (T_r^\ell \hat{q}) \right) \right]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{R_d T_a^\ell} \mathcal{L}^* (\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{R_d T_r^\ell}{C_{vd}} [D + d]$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_{pd}}{C_{vd}} [D - (1 - \kappa) \mathcal{S}^* (D) + d]$$

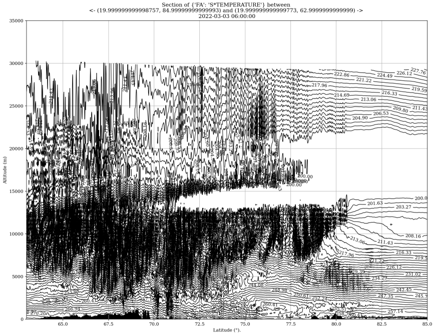
$$\frac{\partial \mathrm{l}_s}{\partial t} = -\mathcal{N}^* (D)$$

- The formulation of $T_r^\ell(t)$, $T_a^\ell(t)$ and T_r^s results from theoretical SI analyses, but also from empirical knowledges on the behaviour of Constant-coefficient SI schemes for mass-based EE system.
- SI parameters can be updated at each timestep (Cheap option) or possibly at each PC iterations (Full option).
- $\pi_{sr}(t)$ is still set to 900hpa as in the current SI_CST approach.

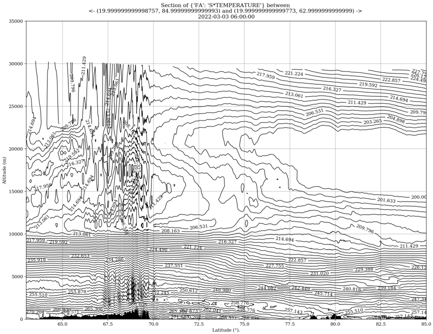
Svalbard problematic run with PC scheme

AROME (1250m, L90, 50s) Temperature after 30H forecast :

PC + SI_CST



PC + SI_UPDATE

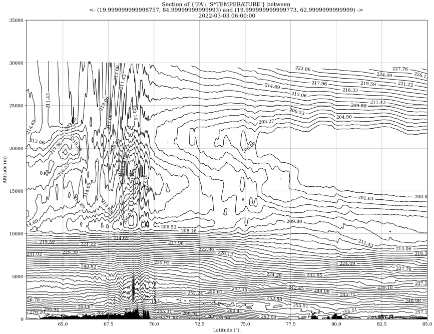


- Updating SI vertical parameters at each timestep with $\beta_r = 0.5$ and $\beta_a = 0.7$.

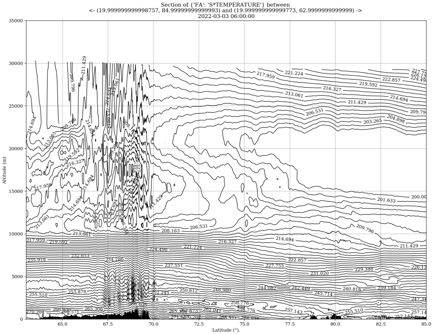
Svalbard problematic run with PC scheme

AROME (1250m, L90, 50s) Temperature after 30H forecast :

NSITER=2



PC + SI_UPDATE



- PC + SI_UPDATE and (NSITER=2) solutions are comparable.

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Introduction of constant SI parameter for orography Slope

Basic underlying idea:

- Take into account the effect of slope in the vertical Laplacian-like operator definition $\mathcal{L}^*$:

$$\mathcal{L}^*(\hat{q}) = \frac{\pi_r^*}{\partial_\eta \pi_r^*}\partial \left[\left\{ \frac{\partial_\eta \left(\pi_r^* \hat{q}\right)}{\partial_\eta \pi_r^*} \right\} + \overline{\frac{\nabla\phi^{*2}}{g^2} \overline{\left\{ \frac{\partial_\eta \left(\pi_r^* \hat{q}\right)}{\partial_\eta \pi_r^*} \right\}}^{\mathrm{v}}}^{\mathrm{w}} \right]$$

with

$$\nabla\phi^* = g\Lambda_s^* + \int_\eta^1 T_{max}^{\ell}[\nabla\delta^*(\Lambda_s^* / T_{min}^{\ell})]\mathrm{d}\eta'$$

- where Λ_s^* (**SISLP**) is an additional constant SI parameter chosen as the maximum of orography slope over the domain ⇒

$$\Lambda_s^* = \max_{x\in\mathcal{D}_h} \left[\sqrt{\left(\partial_x z_s\right)^2 + \left(\partial_y z_s\right)^2} \right]$$

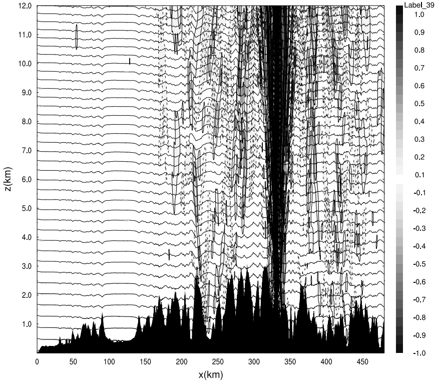
- vertical interpolating operators $\overline{(\,)}^{\mathrm{w}}$ and $\overline{(\,)}^{\mathrm{v}}$ are build in such a way that $\mathcal{L}^*$ akin to a negative definite vertical Laplacian operator.

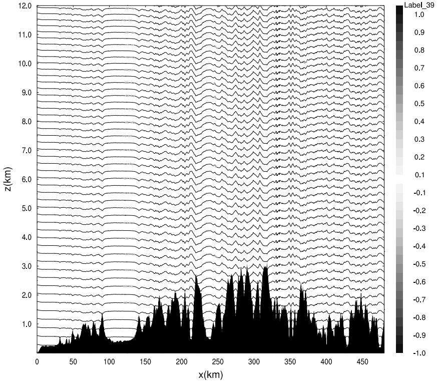
No-Flow test over the Alps

AROME 3D (500m, L90, 25s) academic PC runs,
vertical velocity w, and $iso - \theta$ contours solutions after 24H forecast

SI_UPDATE

SI_UPDATE + SISLP





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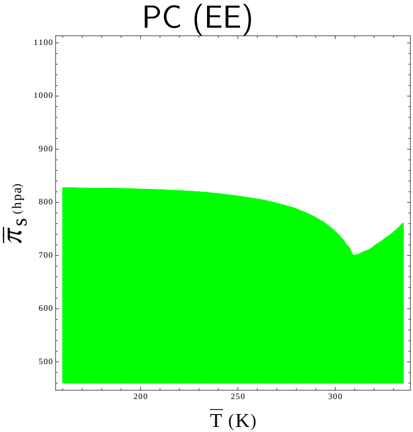
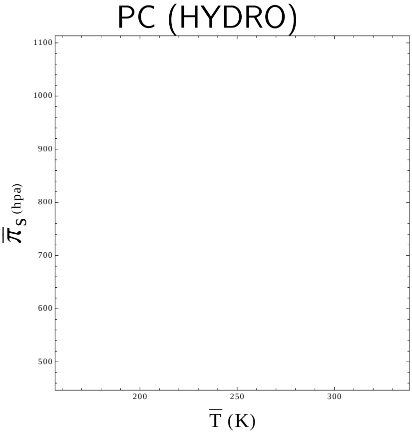
Stability analyses with current SI_CST parameters

SITR=350K, **SITRA**=100K, **SIPR**=900hpa





- **Bad news** : EE with PC exhibits a wide region where Baric NL residuals are detrimental for stability $\Rightarrow$ PC unstable if ($\pi_s < 800\text{hpa} < \pi^*_{sr}$).
- **Good news** : Hydrostatic with PC does not suffer from the effect of Baric NL residuals $\Rightarrow$ Problem comes from the acoustic part.

Gravity/Acoustic splitting

Linear acoustic part :

$$\frac{\partial D}{\partial t} = -RT^*\nabla^2\left[\hat{q} - (1-\kappa)\mathcal{G}^*(\hat{q})\right]$$
$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_a^*}\mathcal{L}^*(\hat{q})$$
$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v}\left[D - (1-\kappa)\mathcal{S}^*(D) + d\right]$$

- The coupling between the two integral terms in **blue** are responsible for the unstable behaviour of EE sytem. Dropping two of them → PC (Acoustic) stable if

$$\pi_{sr}^* < \pi_s$$

used to build the vertical laplacian operator $\mathcal{L}^*$.

Proposed Solution :

- A new prognostic variable for temperature

$$\widehat{T} = Te^{-\hat{q}}$$

$\Rightarrow$

$$\pi\frac{\partial \phi}{\partial \pi} = -R\widehat{T}$$

- Introcduction of a constant low SI acoustic surface pressure **SIPRA**:

$$\pi_{sa}^t = \min_{x\in\mathcal{D}_h} \pi_s(x,t)$$

affecting only the design of the vertical Laplacian operator $\mathcal{L}^* \to \mathcal{L}^*$.

New SI linear EE system with $\widehat{T}$:

$$\frac{\partial D}{\partial t} = -R_d \nabla^2 \left[\left(\mathcal{G}^* (\widehat{T}) + T_r^s q_s \right) + T_r^\ell \hat{q} \right]$$

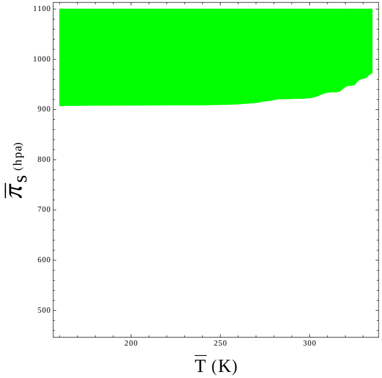
$$\frac{\partial d}{\partial t} = -\frac{g^2}{R_d T_a^\ell} \mathcal{L}^* (\hat{q})$$

$$\frac{\partial \widehat{T}}{\partial t} = T_r^\ell \left[D + d - \mathcal{S}^*(D) \right]$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_{pd}}{C_{vd}} \left[D - (1-\kappa) \mathcal{S}^*(D) + d \right]$$

$$\frac{\partial \mathrm{l}_s}{\partial t} = -\mathcal{N}^*(D)$$

SIPR= 900hpa, **SIPRA**= 450hpa

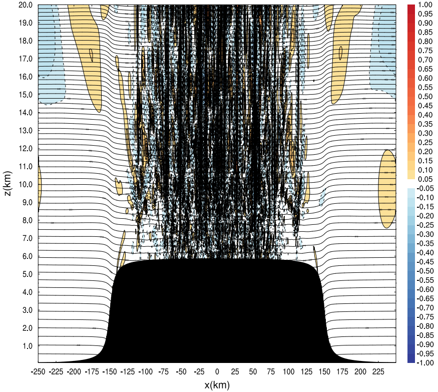


⇒**SIPR**= 1100hpa and **SIPRA**= 450hpa fully stabilize the situation.

3D No-Flow test over a prescribed high plateau, $H_{max} = 6000m$, $S_{max} = 19^{\circ}$

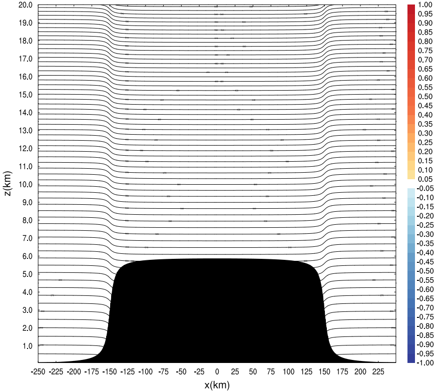
AROME (500m, L137, 15s, NSITER=1) vertical velocity w solution :

6H forecast



NO SIPRA

24h forecast



SIPRA $= 450\,$hpa

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Summary and perspectives

- SI_UPDATE + SISLP strategy offer the possibility to better control the impact of the NL residual terms on stability for the thermal Orographic and Baric NL residuals and therefore to **enhance the robustness of PC scheme** at reasonable cost.
- Strategy need to be intensively tested in more challenging realistic cases (over HImalaya domain for instance).

Thanks for listening !!!